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ORBITAL LAUNCH

ESCAPE WINDOW ANALYSIS

NSL 62-57

NORTHROP SPACE LABORATORIES

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ORBITAL LAUNCH OPERATIONS

ESCAPE WINDOW ANALYSIS

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ABSTRACT

The analysis of the gross aspects of the flight mechanics for lunar trajectories from orbital launch is presented. An attempt is made to define the launch requirements from an earth orbit, the geometrical relationships existing between an Orbital Launch Facility in earth orbit and the moon in its orbit, the departure trajectory sensitivity to energy and time requirements, the launch-on-time problem and the orbital "escape window".

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LIST OF SYMBOLS

i	=	OLF Orbit plane inclination with equatorial plane, deg.
η	=	Lunar plane inclination with equatorial plane, deg.
ϕ	=	Inclination angle between orbit plane and lunar plane, deg.
λ_N	=	Longitude of orbit plane - equatorial plane line of nodes measured counterclockwise from lunar descending equatorial node, degrees.
$\overline{L}$	=	Position vector of vehicle at launch (from geocenter).
$\overline{S}$	=	Position vector of moon at arrival (from geocenter)
β	=	In-plane central angle of transfer trajectory between $\overline{L}$ and $\overline{S}$, degrees.
μ_L	=	Plane change angle between orbit plane and transfer trajectory plane, degrees.
α_L	=	Right ascension of the Vehicle Position Vector projection on lunar plane measured counterclockwise from the lunar ascending node, degrees.
α_S	=	Right ascension of the Moon Position Vector measured counterclockwise from the lunar ascending node, degrees.
δ_L	=	Declination of the Vehicle Position Vector, degrees.
Ω	=	OLF Orbit Plane Precession Rate, degrees/day.
h	=	OLF Orbit altitude, nautical miles.
ΔV	=	Velocity increment, feet per sec.
ΔV_{ADD}	=	Additional velocity increment or penalty, feet per sec.
V_{BO}	=	Burnout Velocity, feet per second.
T_b	=	Trip time, hours.
γ	=	Flight path angle, degrees.
$\hat{\theta}$	=	True anomaly, degrees.
a	=	Semi-major axis of transfer ellipse.
e	=	Eccentricity of transfer ellipse.
Σ_L	=	Launch azimuth, degrees.
$\frac{N}{m}$	=	Rendezvous compatible orbit synchronous ratio.
q	=	Reference parameter equal to the difference between i and η
Δ	=	OLF Plane - Lunar Plane node line position angle measured in the Lunar Plane, from the equatorial plane - lunar plane node line.

INTRODUCTION

This report covers a six month study effort by the Astro Sciences Group of the Norair Division, Northrop Corporation, in the area of the flight mechanics problems of orbital launch. The study was initiated in conjunction with an overall manned space systems study program in progress at the Norair Division. The report presents the preliminary results of an analytical investigation into the fundamental aspects of the optimum launch conditions from an earth orbit.

Inasmuch as the precise definition of launch conditions is strongly dependent on a knowledge of the physical conditions of the launch vehicle and the specific requirements of a particular lunar mission, only a gross analysis is presented which can provide a mission analyst with the tools and data to define a performance envelope for the vehicle designer.

The chief objectives of the escape window analysis were:

(a) To define the earth orbit launch criteria for lunar trajectories.
(b) To establish the departure trajectory sensitivities to orbital parameters.
(c) To establish the mission and vehicle sensitivities to time and energy requirements.
(d) To establish the analytical technique for determining optimum launch conditions.
(e) To provide the basis for the preparation of an orbital launch timetable compatible with lunar mission and earth launch requirements.

The study effort results related to each of the objectives are presented in the following sections. Pertinent assumptions are identified and conclusions and recommendations indicated where appropriate. Technical analyses in support of the data presented are included in the report insofar as is feasible. Computer programs have not been included but are referenced.

The scope of the work included the definition of an operational orbit based on previous studies performed at Norair. This orbit was defined by the earth launch site chosen (AMR), the use of the concepts of rendezvous compatibility and minimum orbit maintenance and the altitude limit due to the lower Van Allen Radiation Belt. The orbit thus defined was at an altitude of 263 nautical miles and an inclination of 30° to the equator. Although an analysis of the escape window should include the effects of different altitudes and inclinations, the tremendous and detailed mathematical and graphical work associated with

this investigation for any single orbit precluded their inclusion during the time allotted for this work.

This report was prepared for submittal to the Astronautics Division, Chance Vought Corporation, Dallas, Texas in accordance with agreements between representatives of the Norair and Vought Astronautics, to provide the results of this study in support of the Orbital Launch Operations contract between Chance Vought Corporation and Marshall Space Flight Center (MSFC Contract NAS 8-853).

The format for this report results from instructions from Chance Vought to duplicate the format of the Orbital Launch Operations Progress Report, Volume II, dated 14 June 1961. The discussion has been so designed as to permit extraction of only the technical approach and results for inclusion in an overall report by Chance Vought to Marshall Space Flight Center.

1.0 ESCAPE WINDOW ANALYSIS

1.1 INTRODUCTION

It is the purpose of this section to present the gross aspects of the flight mechanics for lunar trajectories from orbital launch with emphasis upon specifically defining the "escape window". An attempt is made to define the launch requirements from an earth orbit, the geometrical relationships existing between an Orbital Launch Facility (OLF) in earth orbit and the moon in its orbit, the departure trajectory sensitivity to energy and time requirements, and the launch-on-time problem.

1.2 ORBITAL LAUNCH CRITERIA

Launch from orbit is a function of many variables, both geometric and dynamic. A clear understanding of the three dimensional geometric relationship of the satellite-earth-moon system is necessary to visualize the problem. The illustration on Figure 1 is a display which correlates the angular and rotational variables of the three planes of interest.

The angle η between the equatorial and lunar planes varies slowly between 18.5° and 28.5° over an 18.6 year cycle as shown on Figure 2. Due to the relatively slow variation in η when compared to the real time requirements for any single lunar mission, this angle can be considered a constant in the analysis once a launch or mission date is specified. The OLF plane maintains a constant inclination, i, to the equatorial plane as it precesses (due to earth oblateness) around the earth in a retrograde direction (from east to west). This precession rate is a function of OLF altitude and orbit plane inclination as shown in Figure 3. For the chosen operational altitude and inclination (263 nautical miles and 30° inclination), this rate is approximately 6.7° per day. The significance of orbit precession is analyzed in a later paragraph.

The angle ϕ between the OLF and lunar planes is a function of i, η, and the position of the satellite plane at any instant. This position is defined by the equatorial angle λ_n or the longitudinal location of the satellite ascending equatorial node line as measured counterclockwise from the lunar descending equatorial node. The trigonometric relationship of these four angles defines the spatial relationship of the three planes at any time and is stated thus:

$$\phi = \cos^{-1} (\cos i \cos \eta - \sin i \sin \eta \cos \lambda_n)$$

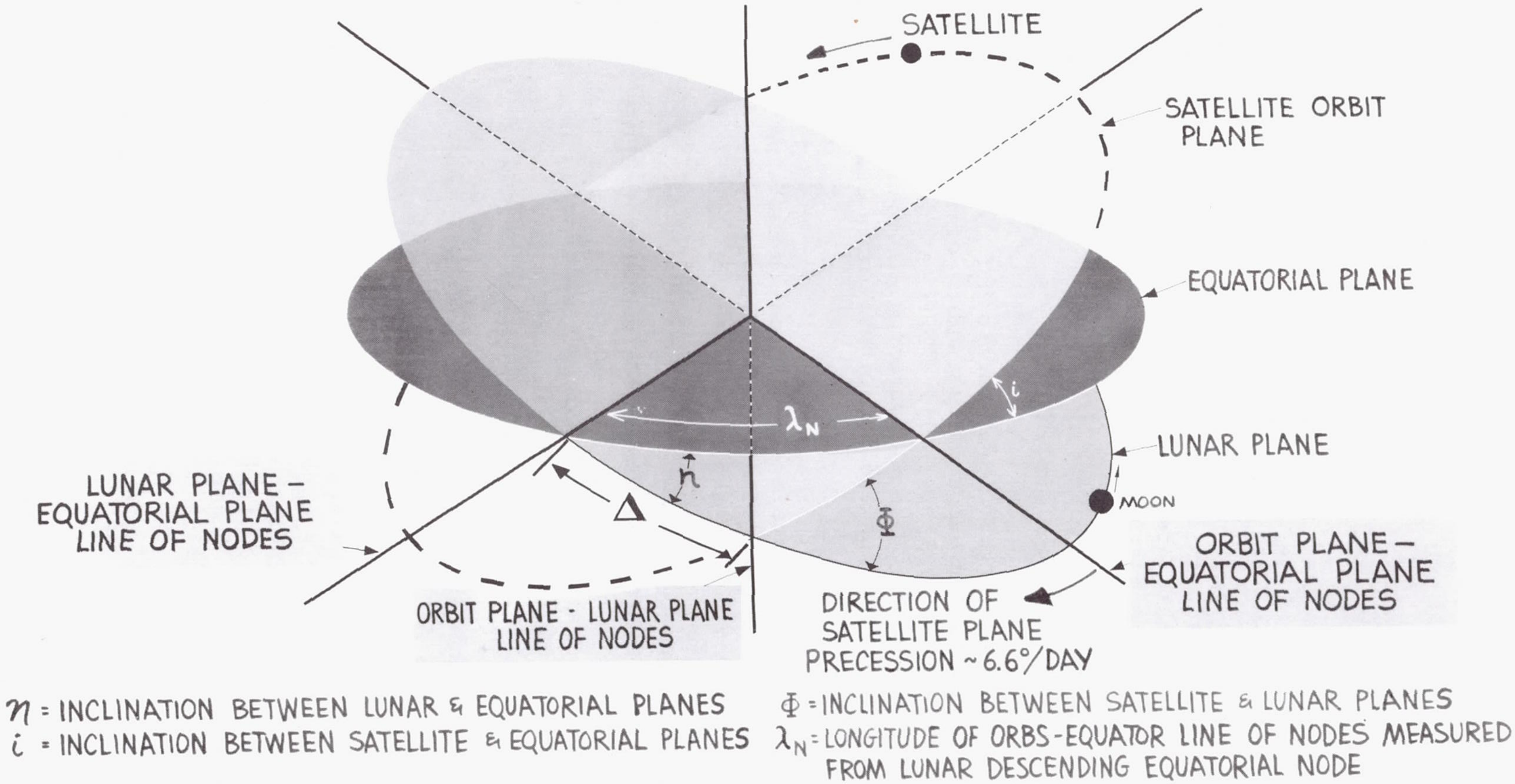


FIGURE 1 GEOMETRIC RELATIONSHIP FOR SATELLITE-EARTH-MOON SYSTEM

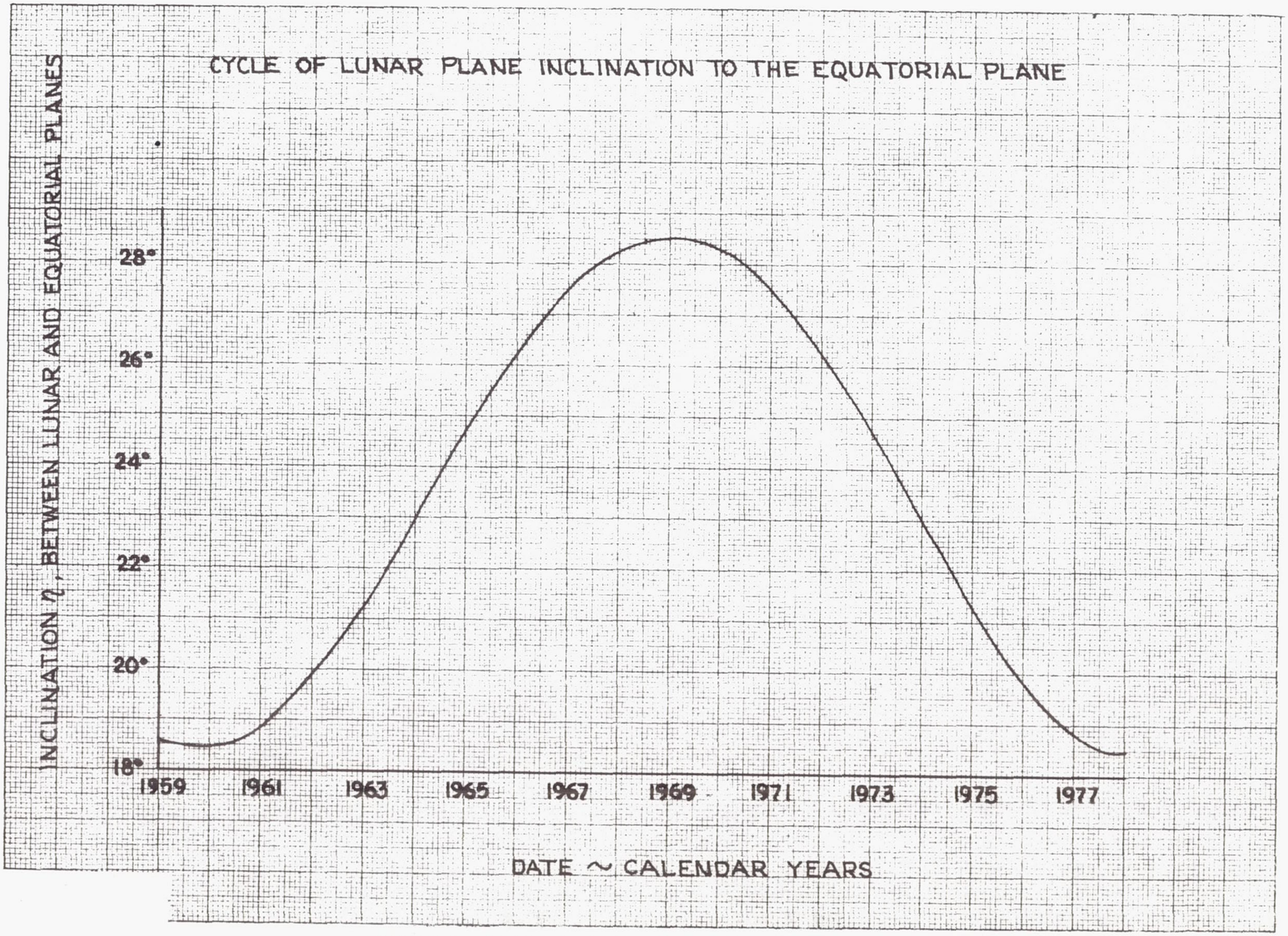


FIGURE 2 LUNAR PLANE INCLINATION CYCLE (η vs t)

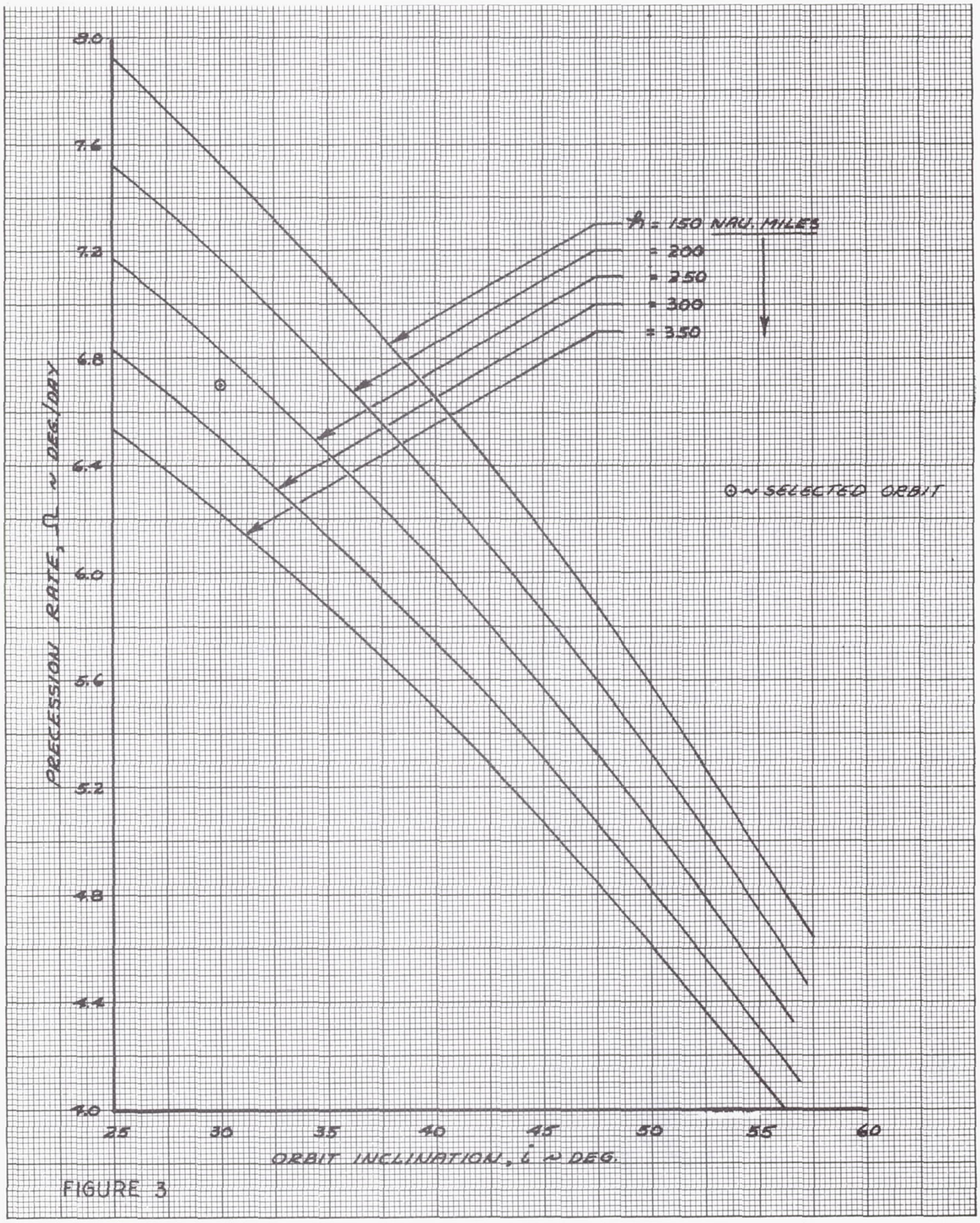


FIGURE 3 ORBIT PLANE PRECESSION RATE

Since λ_n is changing at the precessional rate of the OLF plane (6.7°/day), ϕ will have a 54 day cycle (one complete rotation of the OLF plane). This variation of ϕ is illustrated in Figure 4. The bandwidth for i = 30° illustrates the effect of a two year variation in η. It will be shown later that the launch-on-time restrictions and lunar arrival window are a strong function of the angle ϕ.

1.3 OPTIMUM LAUNCH CONDITIONS

To define the optimum launch conditions, only the positional variables of the OLF and moon are significant. Projection of the OLF orbit, orbital launch vehicle transfer trajectory, and lunar orbit plane onto a geocentric sphere results in the orbital launch geometric parameter relationship shown on Figure 5. A description of the parameters illustrated follows:

$\overline{L}$ = Position vector (from geocenter) of the vehicle at time of launch. Located by the right ascension α_L measured from the ascending node, and the declination to the lunar plane, δ_L.

$\overline{S}$ = Position vector (from geocenter) of the moon at arrival. Located by the right ascension α_S measured from the ascending node.

β = In-plane angle of the transfer trajectory between $\overline{L}$ and $\overline{S}$. This angle is a function of the trip time desired.

ϕ = The inclination angle between the OLF and lunar planes.

It is of interest to correlate the four basic parameters mentioned above, to determine the extent that launches other than in-plane trajectories are feasible, to define launch window size, to discuss the launch-on-time problem, and to determine the penalties of late launch.

In the orbital launch parameters illustration on Figure 5, the out-of-plane launch angle μ_L is determined by the plane of the flight path. This plane, in turn, is determined by the geocenter and the position vectors $\overline{L}$ and $\overline{S}$. If $\overline{L}$ and $\overline{S}$ are treated as unit vectors then

$$\overline{S} = iS_x + jS_y + kS_z$$
$$\overline{L} = iL_x + jL_y + kL_z \qquad (1)$$

where i, j, and k are unit vectors in the x, y, z coordinate system shown on the latter figures. The positive x-axis is in the direction of the ascending node of the OLF plane with respect to the lunar plane.

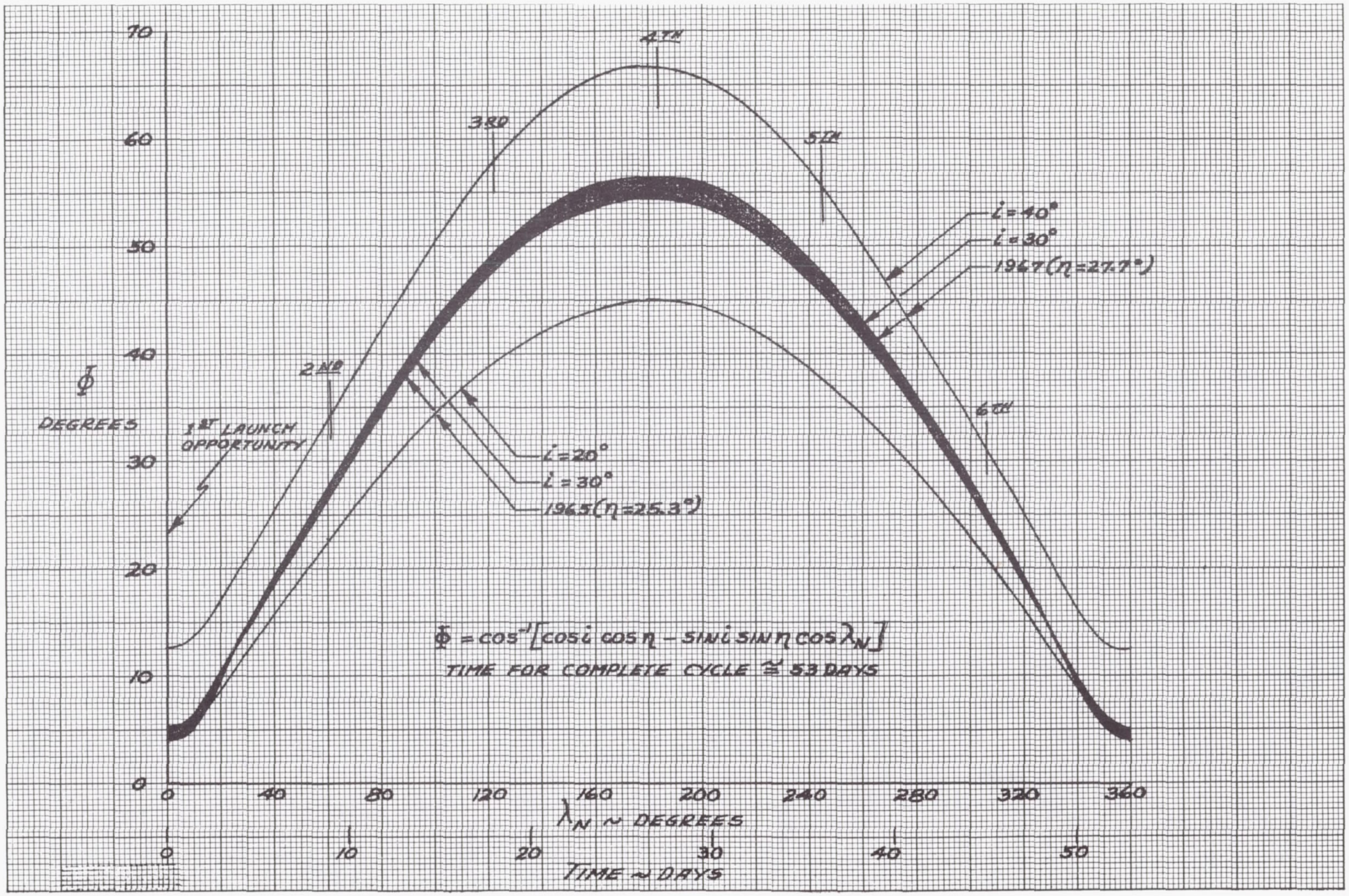


FIGURE 4 ORBIT PLANE — LUNAR PLANE INCLINATION CYCLE (ϕ vs λ_N)

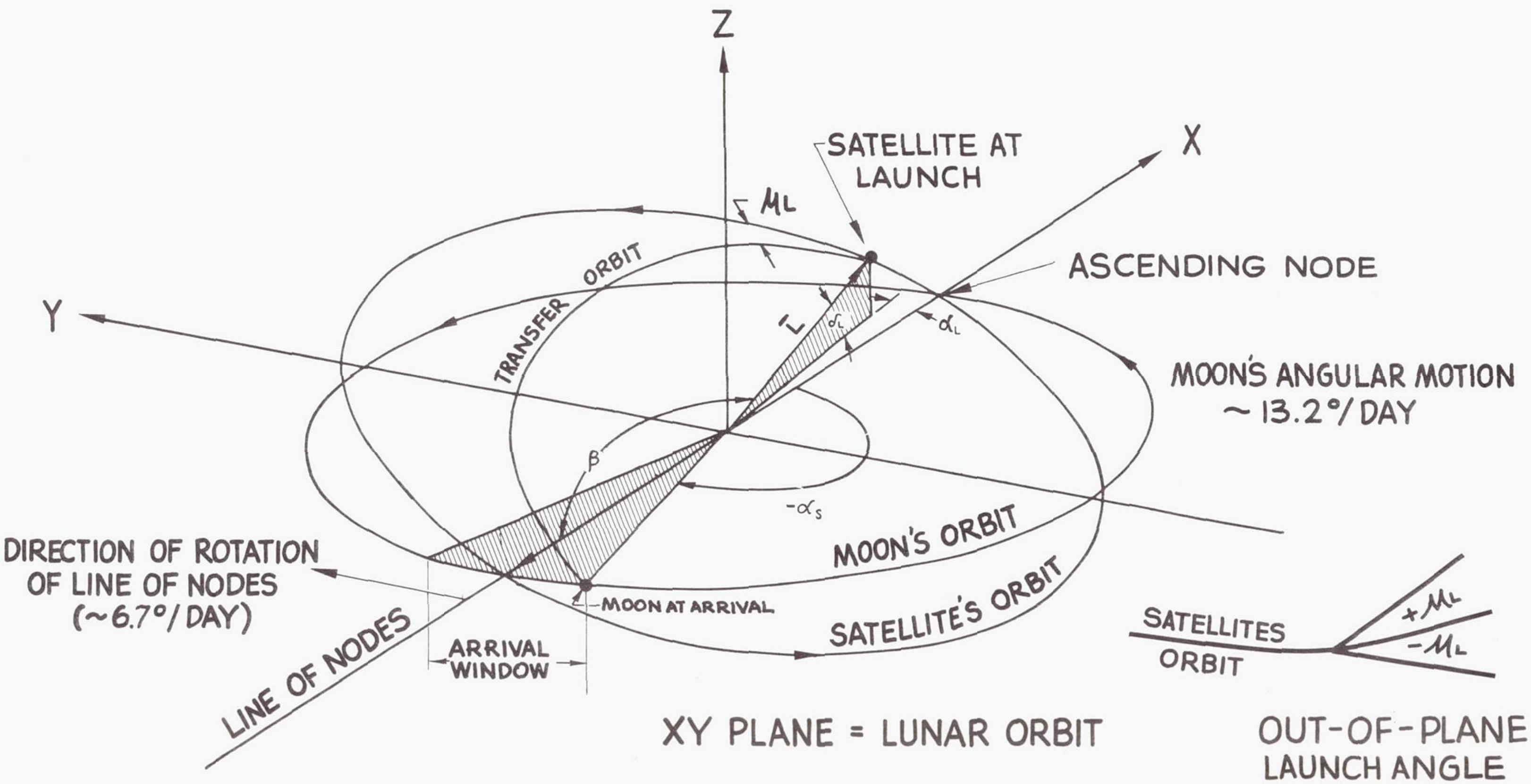


FIGURE 5 ORBITAL LAUNCH GEOMETRIC PARAMETERS

The x-y plane lies in the lunar plane. In such a system:

$$\left.\begin{aligned} S_x &= \cos \alpha_s \\ S_y &= \sin \alpha_s \\ S_z &= 0 \end{aligned}\right\} \qquad (2)$$

$$\left.\begin{aligned} L_x &= \cos \alpha_L \cos \delta_L \\ L_y &= \sin \alpha_L \cos \delta_L \\ L_z &= \sin \delta_L \end{aligned}\right\} \qquad (3)$$

where α_s = right ascension of moon at arrival

$\left.\begin{aligned} \alpha_L &= \text{right ascension of OLF} \\ \delta_L &= \text{declination of OLF} \end{aligned}\right\}$ at launch

By taking the dot product of $\overline{L}$ and $\overline{S}$, the following relationship is obtained

$$\beta = \text{arc cos} \left[\cos \delta_L \cos (\alpha_S - \alpha_L) \right] \qquad (4)$$

From spherical trigonometry

$$\tan \delta_L = \frac{\sin \alpha_L}{\cot \phi} \qquad (5)$$

$$\sin \mu_L = \frac{\sin \alpha_S \sin i}{\sin \beta} \qquad (6)$$

For a specific ϕ, and with α_s and α_L as variables, calculations based upon the above relationships will yield the entire spectrum of possible trajectories between the OLF circular orbit and the lunar orbit. For an OLF plane inclination, i = 30°, and a lunar plane inclination, η = 23° (this η is representative of the 1964 time period), ϕ will range between 53° and 7° over its 54 day cycle. For discussion and illustration, these boundary values of ϕ (53° and 7°) are chosen as typical and are used for more detailed analysis.

The entire spectrum for ϕ = 53° and 7° is displayed in Figures 6 and 7 where the central angle and plane change requirement can be determined for any position of the moon at arrival and satellite at launch. On these figures are superimposed lines of constant central angle β (light lines) and constant plane change μ_L (heavy lines). The $+\mu_L$ and $-\mu_L$ lines represent plane changes to the left and right of the

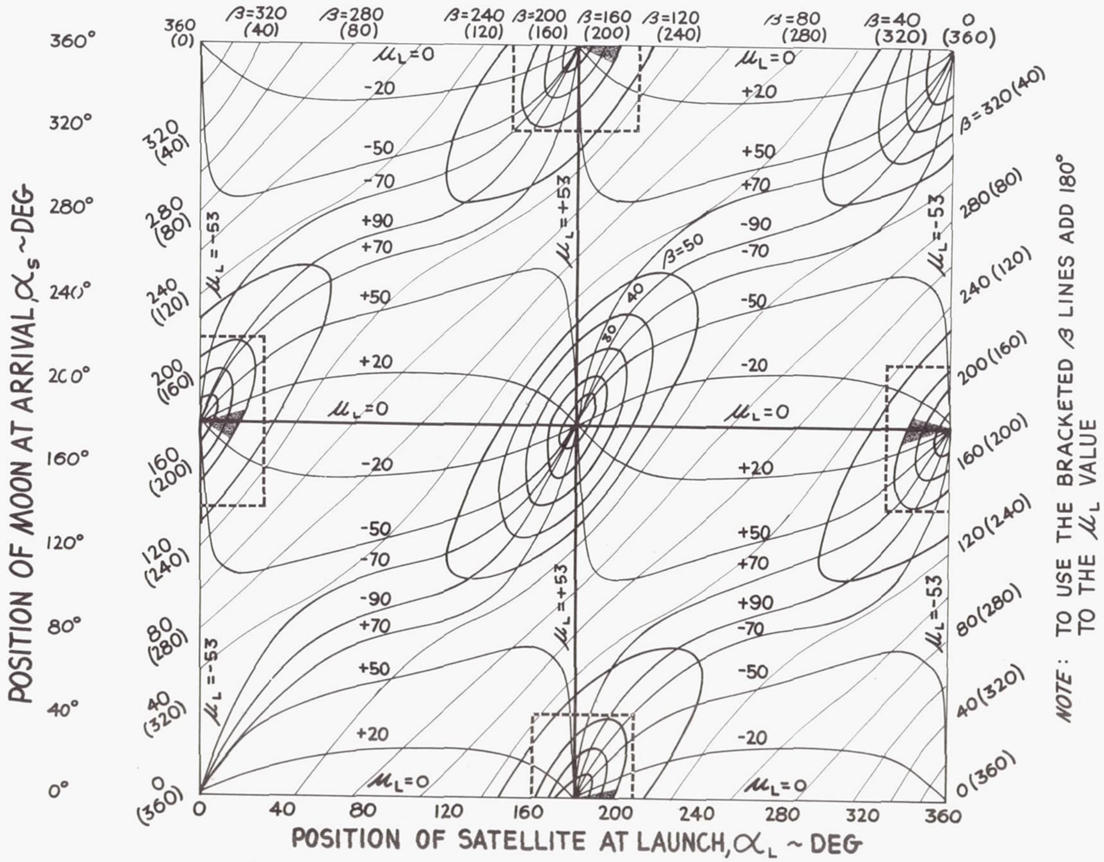


FIGURE 6 COMPLETE DISPLAY OF GEOMETRIC PARAMETERS, ϕ = 53°

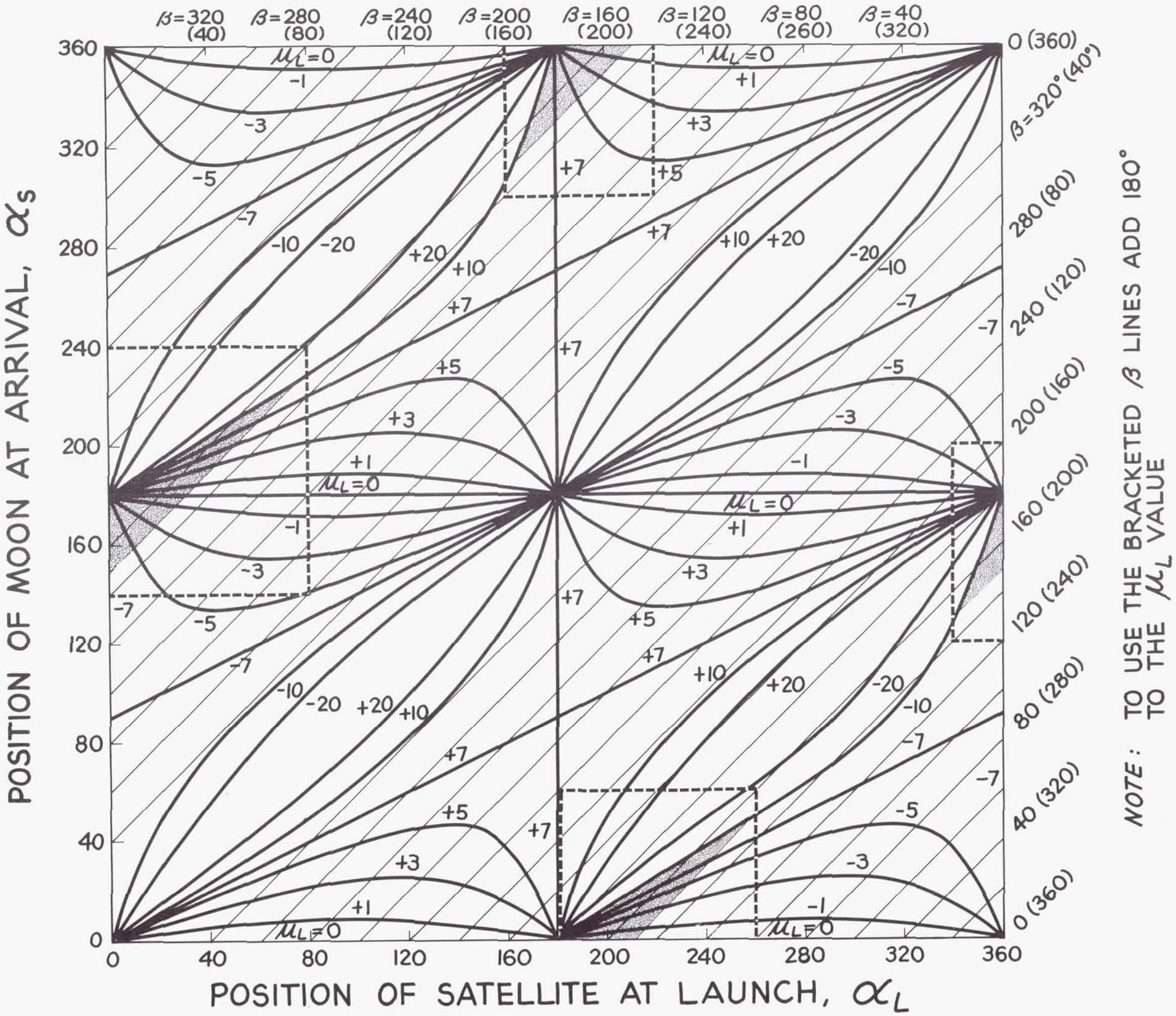


FIGURE 7 COMPLETE DISPLAY OF GEOMETRIC PARAMETERS, Φ = 7°

OLF orbit plane. Each point on the α_S vs α_L grid defines a unique pair of transfer paths. One path (the indicated μ_L and unbracketed β's) requires a $\mu_L \leq 90°$. The alternate route is the explement of the latter path and requires a launch "backwards" ($\mu_L > 90°$). The backward paths are obtained from Figures 6 and 7 by applying the bracketed values of β and adding 180° to each μ_L line. The $\mu_L > 90°$ trajectories are certainly impractical and are pointed out merely for interest. Figures 8 through 14 are included to show the effect of ϕ upon the geometrical relationship between α_S, α_L, and μ_L. Only practical β's are shown.

The optimum launch time for a specified central angle occurs when the vehicle can launch with a minimum amount of expended energy. Since a plane change at the time of launch requires extra energy, the optimum launch situation exists if the transfer trajectory remains in the plane of the OLF ($\mu_L = 0$) thus taking maximum advantage of the orbiting velocity of the OLF. With the transfer orbit plane and the OLF plane coplanar, the vehicle will arrive at the moon at the line of nodes of the OLF and lunar planes. Therefore, the ideal transfer trajectory for a given central angle occurs when the moon is at either node (i.e., α_S is 0° or 180°) at vehicle arrival. Since the moon is rotating in its orbit at a mean rate of 13.2°/day, and the OLF plane is precessing an average of 6.7°/day in a direction opposite to the lunar motion, the moon is moving "effectively"* 19.9°/day relative to the OLF - lunar plane node line. At this rate, an optimum launch situation presents itself every $\frac{180}{19.9} = 9.05$ days which is the time required for the moon to travel from the ascending node to the descending node using a linear average rotation rate for the OLF plane-lunar plane node line. For gross determinations of the cyclic occurence of the launch window, the linear rate is useful. A discussion of the effect of introducing the non-linear motion of the node line is contained in paragraph 1.6 and must be considered in any accurate determination of the interval between escape windows.

Figure 15 shows schematically the mechanics of launch window occurrence for each time the moon crosses the node line. This figure shows the first window occurring when the moon is at First Quarter (for illustration purposes only). The size of the launch window is determined by the value of ϕ at the time of launch. Specification of ϕ for any i, h, or η determines the rotational rate of the node line. The second window occurs when the counterclockwise motion of the moon and the clockwise movement of the node line causes the moon to again intersect the line of the nodes. The size of the second window differs from the first due to the changing nature of ϕ during the interval between windows. The dependence of window size upon ϕ will be described in a later paragraph.

* "Effectively" is used because a specific determination of the relative motion between the moon and the line of nodes would have to account for (1) what portion of its elliptical orbit the moon is in, (2) the inclination ϕ, (3) the sinusoidal variation in satellite precession rate, (4) the non-linear rotation rate of the node line between the OLF and lunar planes, which has a significant effect on the launch window cycle.

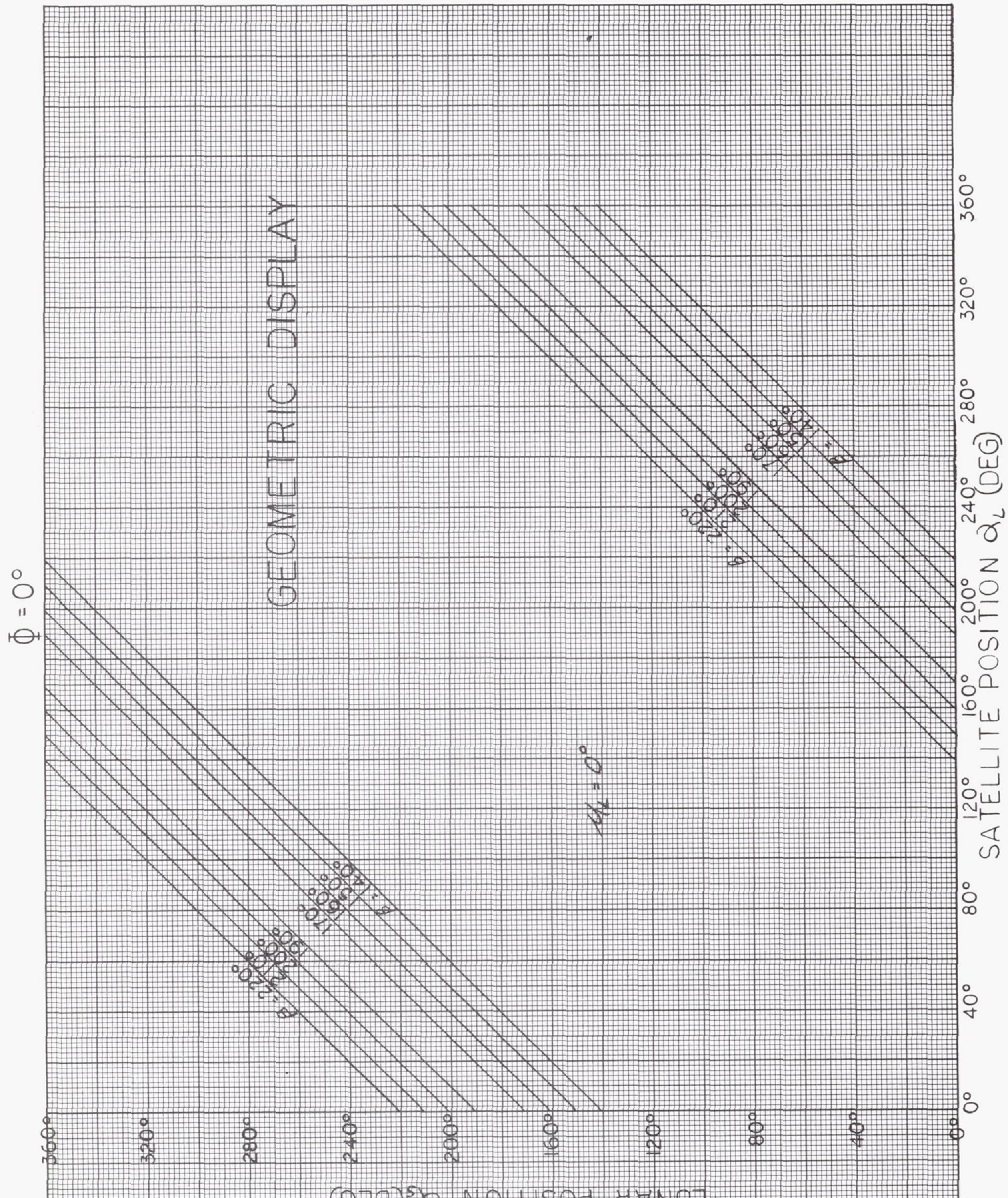


FIGURE 8 GENERAL GEOMETRY DISPLAY $\phi = 0°$

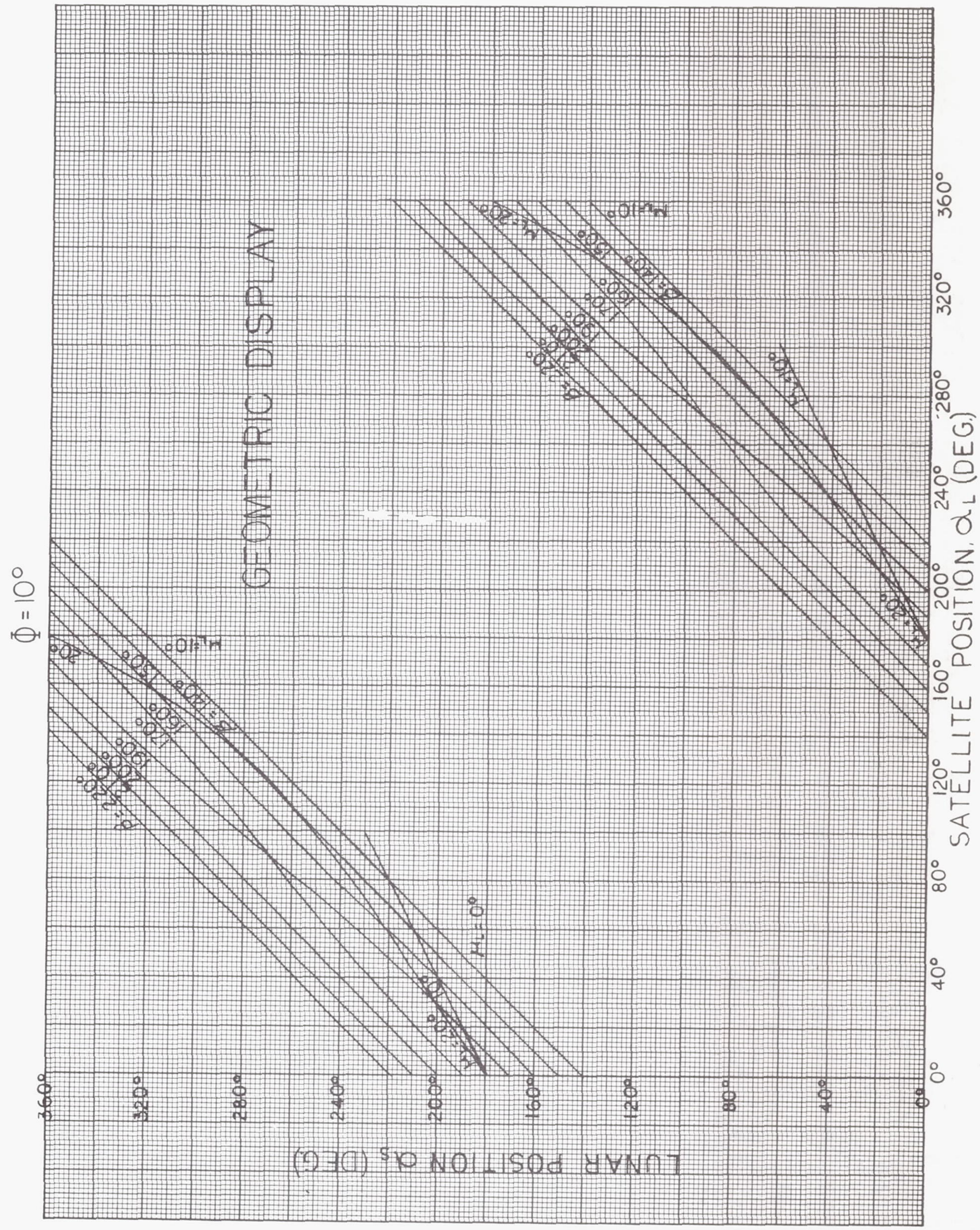


FIGURE 9 GENERAL GEOMETRY DISPLAY ϕ = 10°

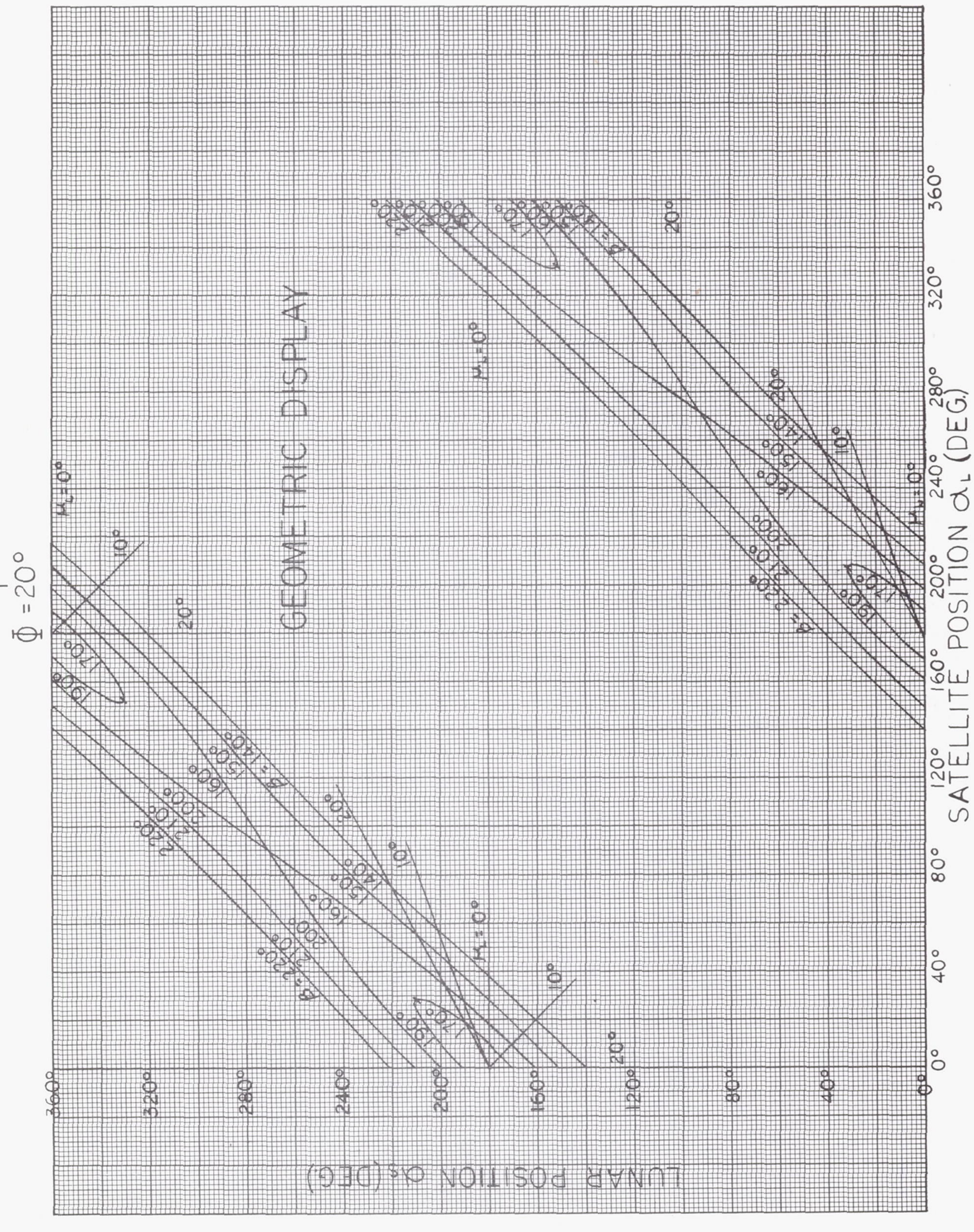


FIGURE 10 GENERAL GEOMETRY DISPLAY $\phi = 20°$

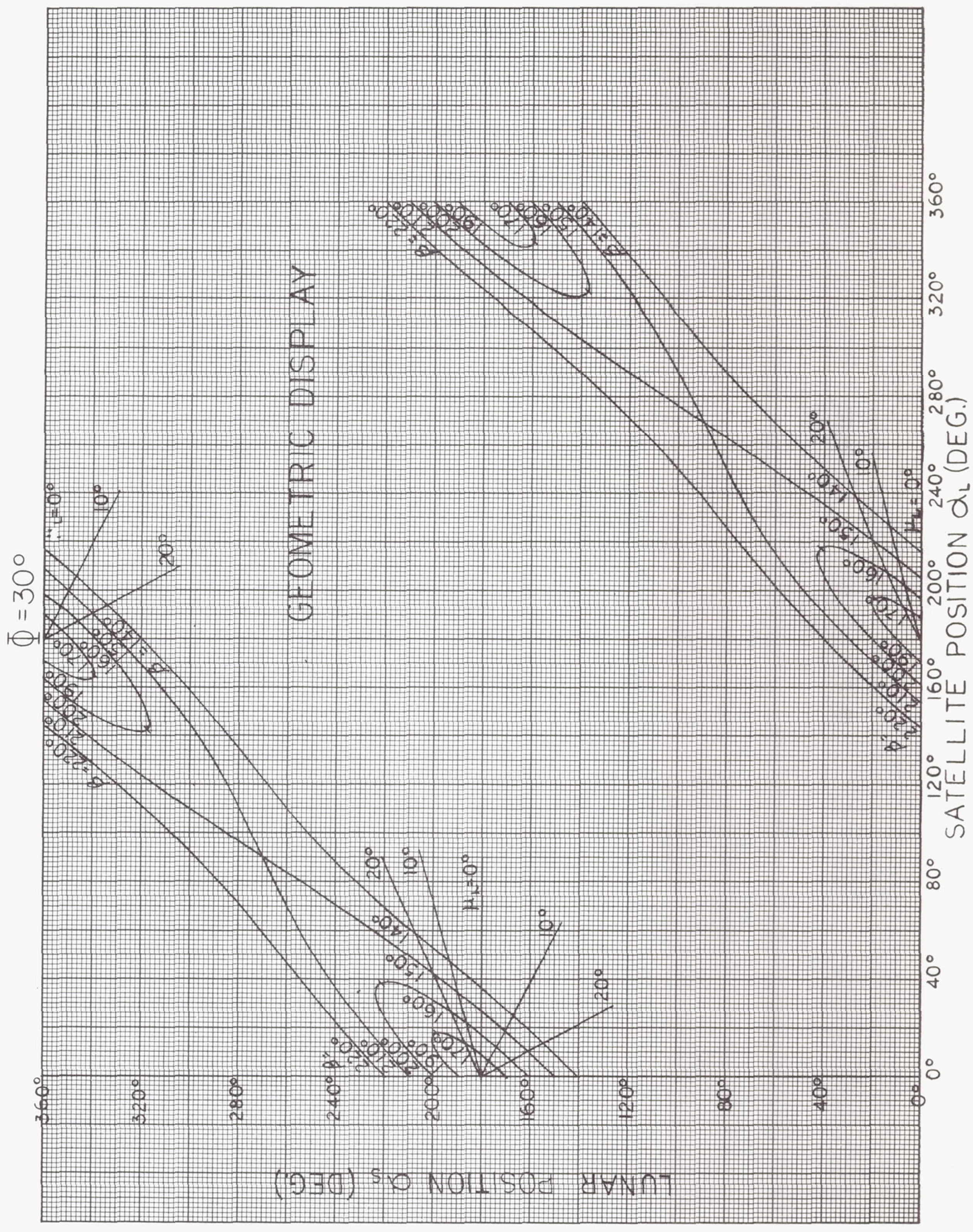


FIGURE 11 GENERAL GEOMETRY DISPLAY $\phi = 30°$

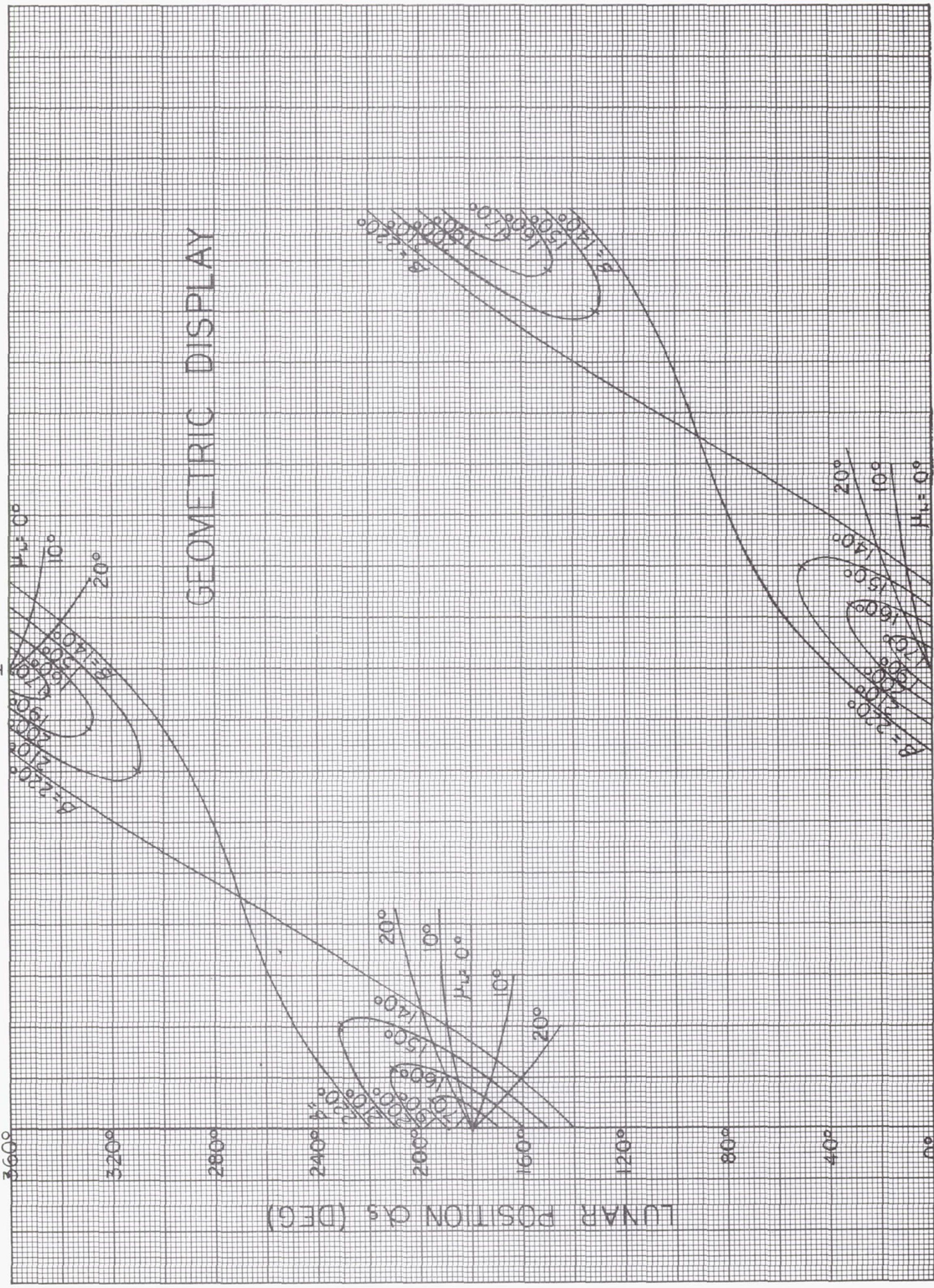


FIGURE 12 GENERAL GEOMETRY DISPLAY $\phi = 40°$

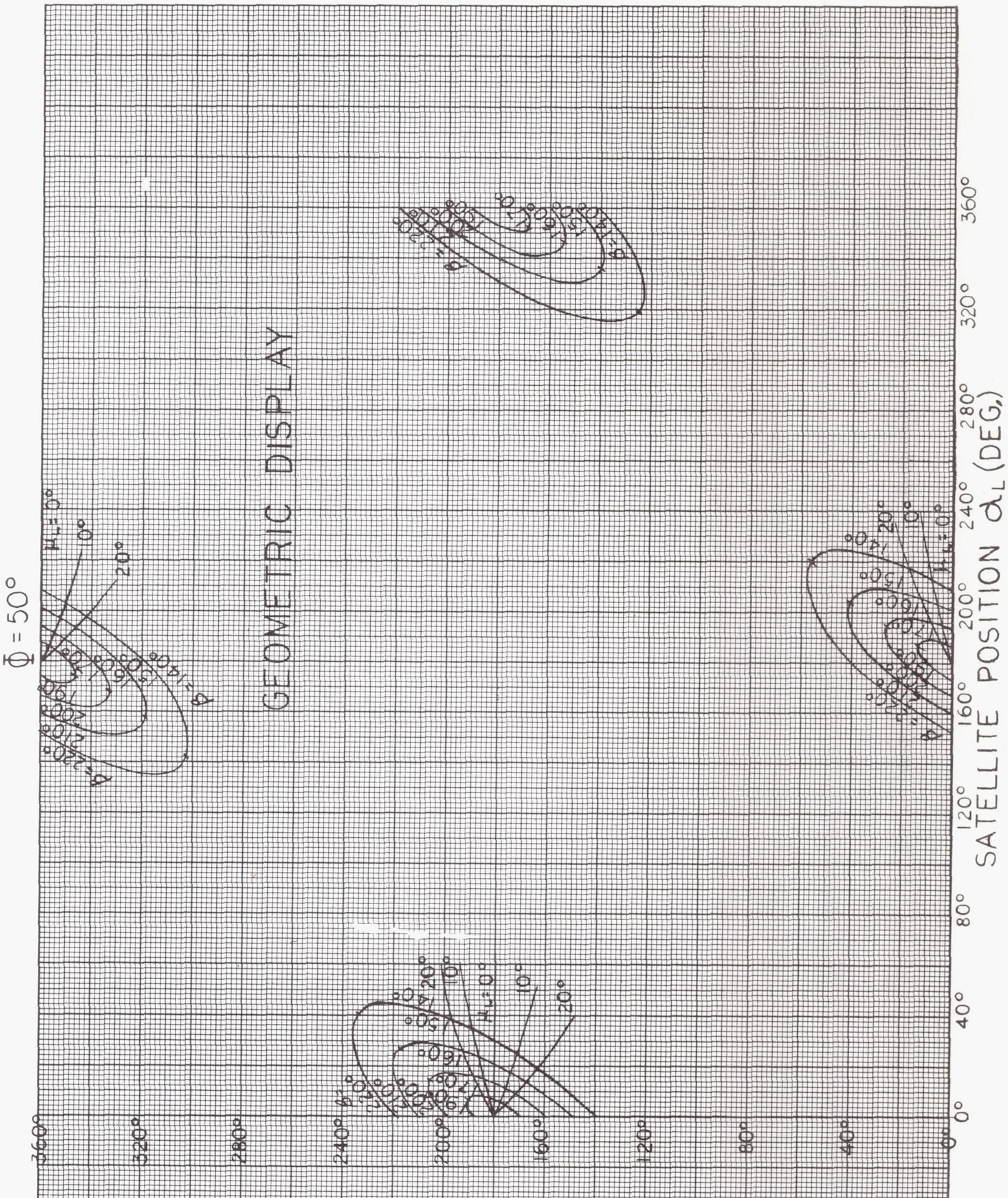


FIGURE 13 GENERAL GEOMETRY DISPLAY $\phi = 50°$

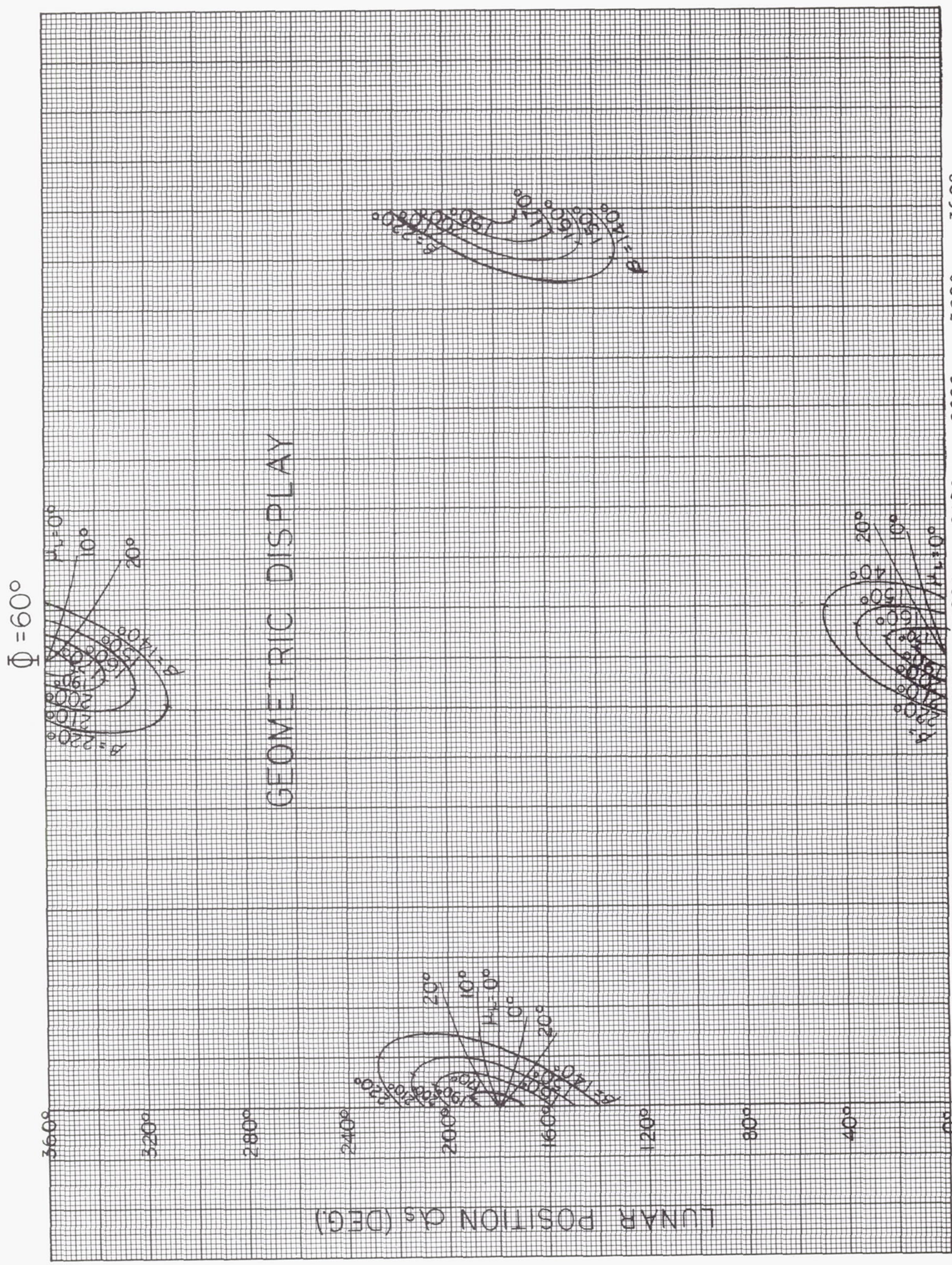


FIGURE 14 GENERAL GEOMETRY DISPLAY ϕ = 60°

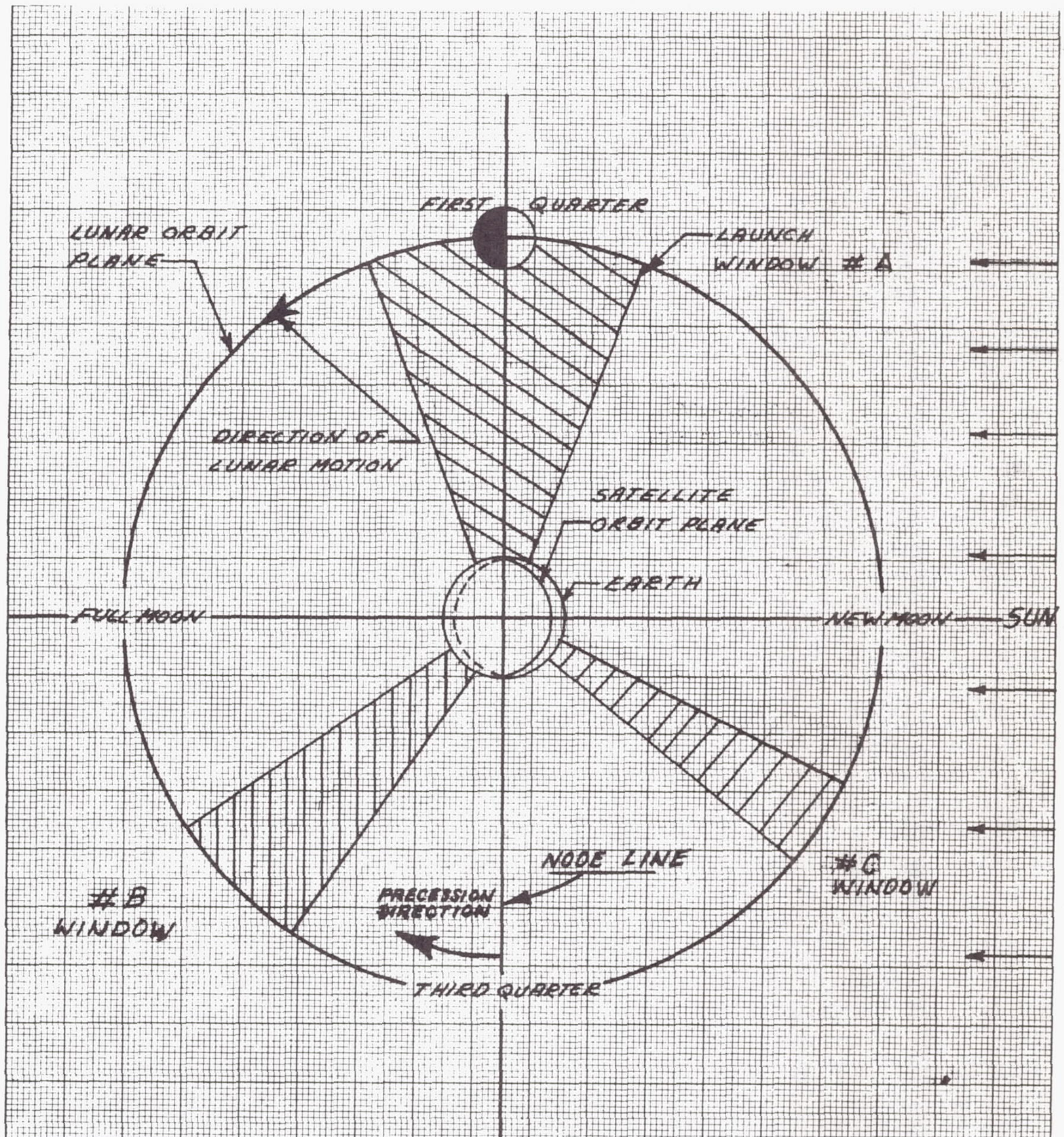


FIGURE 15 LUNAR PHASE RELATIONSHIP

1.4 TRAJECTORY ANALYSIS

1.4.1 Description of Computer Program

The digital computer program discussed in Reference (1) was the prime source of automatic trajectory computation for the included analysis. This program is based upon a modified Buckheim model of the earth-moon system (Reference 2) which ignores the following perturbing forces:

(a) Effect of the Sun and other planets
(b) Solar radiation pressure
(c) Eccentricity of the moon's orbit
(d) Oblateness of the earth and the moon

The effects of these perturbations upon a lunar trajectory are listed:

(a) The presence of the Sun can alter the required burnout velocity as much as 10 ft/sec.

(b) The effect of radiation pressure is less than that of the Sun.

(c) The velocity difference between launching to the moon when it is at apogee from the value at perigee is a 50 ft/sec increment.

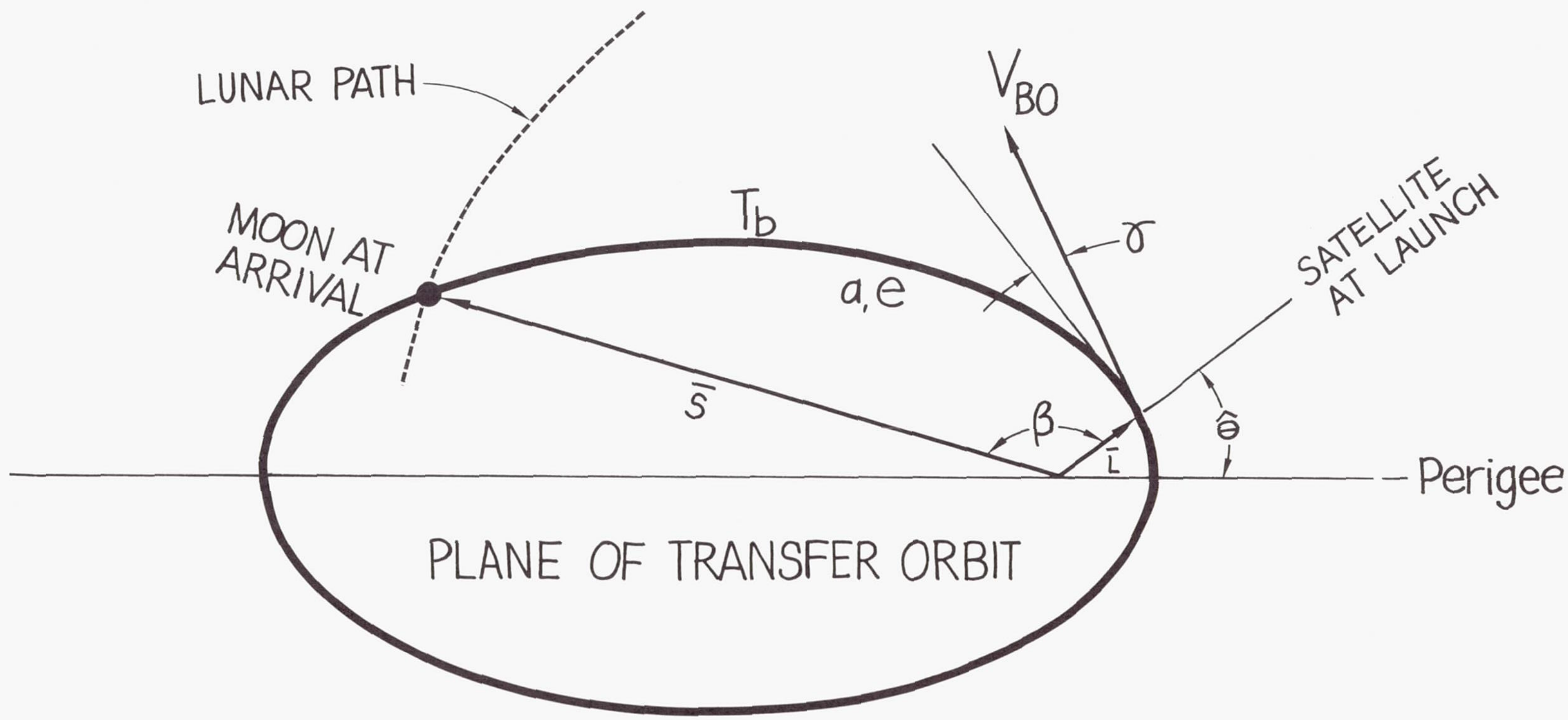
(d) A precise figure for the perturbing effect of oblateness was not determined; however, it is of a magnitude comparable to the Sun's influence.

For the purpose of this analysis, interest lies mainly in general analysis, and comparisons of various trajectories. The restricted three body program is adequate (and in some cases desirable) for this type of computation. Such a simplified program is faster and less complex to operate than the more sophisticated routines. Calculations were performed on an IBM 7090 computer.

1.4.2 Two-Dimensional Analysis

Using the above mentioned IBM Program a two-dimensional analysis of the trajectory conic was performed in the plane of the moon orbit to determine the relationship between the various in-plane dynamic parameters displayed on Figure 16. The results are shown on Figure 17 where the trip time, central angle, launch burnout velocity, and vehicle arrival velocity at pericynthion* are correlated. The position of the moon at launch was timed such that the pericynthion altitude of the transfer trajectory was 50 nautical miles and pericynthion velocity could be correlated with launch burnout velocity. Launch altitude for this study was 263 nautical miles and burnout flight path angle was specified as zero (burnout at perigee of the transfer conic). The information on Figure 17 was then applied to determine in-plane data for all trajectories in this report regardless of their inclination to the equator. This assumption is within the accuracy of the computer program

* Pericynthion - The point of closest approach of a trajectory to the lunar surface



LEGEND:

$\bar{S}$ = POSITION VECTOR OF MOON AT ARRIVAL
$\bar{L}$ = POSITION VECTOR OF OLF AT LAUNCH
V_{BO} = BURNOUT VELOCITY
γ = BURNOUT FLIGHT PATH ANGLE
β = CENTRAL ANGLE
$\hat{\theta}$ = ANGULAR DEVIATION OF LAUNCH POINT FROM PERIGEE–TRUE ANOMALY
T_b = TRIP TIME

FIGURE 16 ORBITAL LAUNCH DYNAMIC PARAMETERS

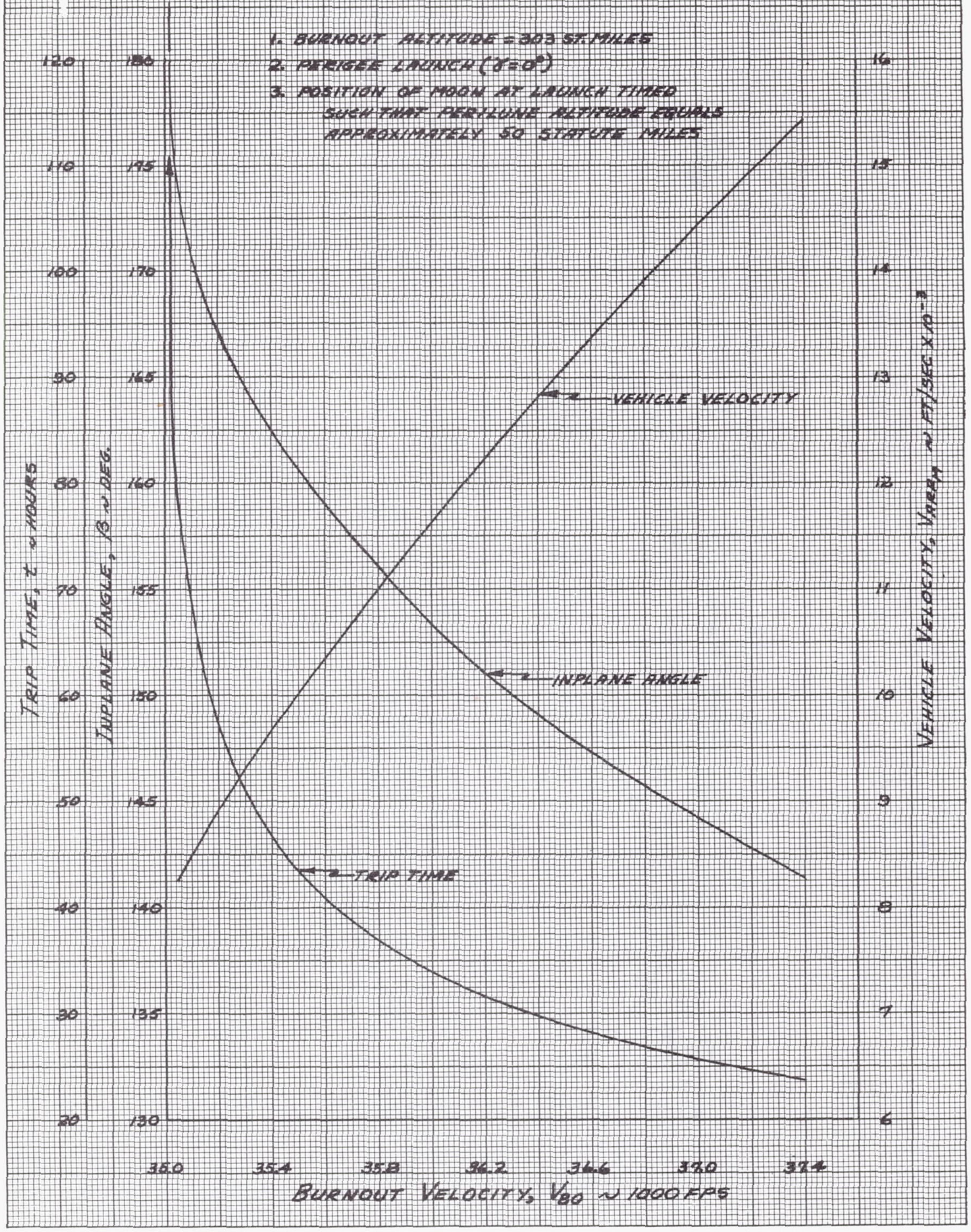


FIGURE 17 DYNAMIC PARAMETERS DISPLAY

employed. Figure 17 indicates that central angle and trip time increase as burnout velocity decreases. As β approaches V_{BO} minimum (β = 180°), the velocity reduction becomes insignificant compared to the increased trip time.

If circular orbit about the moon is desired, an incremental retrothrust must be imposed on the pericynthion velocity in order to achieve lunar circular orbit velocity (5380 ft/sec) at the 50 nautical mile altitude. The velocity increment described by the equation below includes the retrothrust requirement at the moon plus the in-plane velocity which must be added to the circular orbit velocity at earth to launch the transfer trajectory.

$$\Delta V = (V_{BO} - V_{circ_e}) + (V_{P_m} - V_{circ_m})$$

If the moon at arrival is not at the node line, then a plane change angle, μ_L, must be negotiated at the time of launch to accomplish lunar rendezvous. The velocity penalty which a given μ_L imposes is described in Figure 18. The effect of central angle and altitude upon this curve is negligible for the practical range of these parameters. Thus an in-plane angle β requires the V_{BO} described in Figure 17; if the plane of the trajectory is inclined to the OLF plane by μ_L, then an additional velocity (Figure 18) must be added to obtain the total energy requirement.

1.5 ESCAPE WINDOW

It has been indicated that launch should occur when the resulting transfer trajectory can cause rendezvous with the moon at the line of nodes. If launch does not occur on time, an adjustment must be made in trajectory central angle β or the launch plane change angle μ_L or both. These remedies require additional energy, the amount of which increases with late time. An area exists (shown shaded in Figure 5) which extends on either side of the line of nodes that can be reached with feasible alterations in the original programmed trajectory. The time required for the moon to pass through this area of feasibility defines what can be termed a geometric window. More important than the geometric window are the adjustments made at the launch point for the purpose of providing a "launch or escape window". A precise determination of the window size or length of time which is available for launch when the moon is in this area must take into account the following:

(a) OLF-lunar plane geometry,
(b) the launch technique, and
(c) the amount of additional energy (called ΔV_{ADD}) which is carried to provide an escape window.

Detailed discussion of each of these items follows.

1.5.1 OLF-Lunar Plane Geometry

Arbitrary limits of $-10° \leq \mu_L \leq +10°$ and $150° \leq \beta \leq 180°$ are applied for definitive and display purposes. The energy or velocity

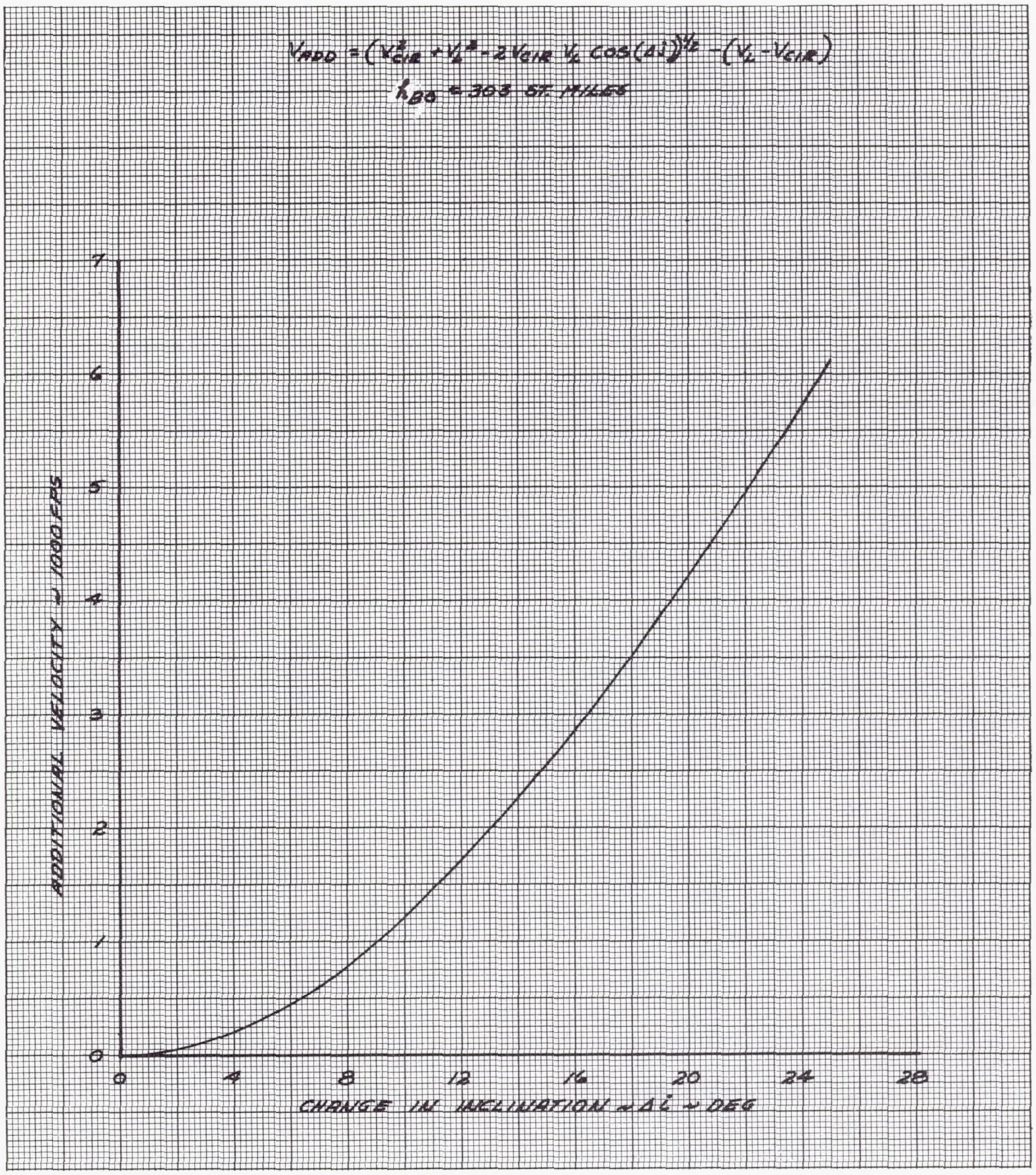


FIGURE 18 PLANE CHANGE VELOCITY INCREMENTS

requirement in this range can be determined by adding the burnout velocity corresponding to a given β (from Figure 17) directly to that ΔV specified by μ_L. The arbitrary limits are seen to be practical limitations also since a 10° plane change requires 1200 ft/sec additional velocity, and a reduction in β from 180° to 150° increases the in-plane burnout velocity 1284 ft/sec. The sum of these two results in approximately a 25 percent velocity increase over a more ideal case (i.e. β = 170°, μ_L = 0°)

The range enclosed by the imposed limitations is shown shaded in Figures 6 and 7 and enlarged in Figures 19 and 20. When the moon is outside this range, launch becomes impractical due to excessive energy requirements. It can be seen in Figure 6 that when $193° < \alpha_s < 347°$ and $13° < \alpha_s < 167°$, transfer trajectories with $\beta < 170°$ are not possible, let alone feasible. It is noticed by comparing Figures 19 and 20 that the launch window increases in size as ϕ decreases and β increases. At ϕ = 53° (Figure 19) the maximum window size, for the average relative motion rate considered, occurs at β = 150°. This window increases considerably for ϕ = 7° under the same energy and plane change limitations, as shown in Figure 20. The maximum window possible occurs when ϕ = 0°, i.e., the OLF and lunar planes are coplanar. This condition cannot occur for i = 30° and η = 23° as used in the illustrative figures. It could possibly occur in 1969 with both i and η equal to 28.5°, and proper positioning of the satellite plane. The effect on window size of the non-linear relative motion rate is discussed in paragraph 1.6.

1.5.2 Launch Technique

Two general approaches that exist for the accomplishment of lunar missions from an OLF are -

(a) Trip time may be held constant (β preselected and fixed)

(b) Trip time may be flexible (β variable and determined by lunar position at the time of launch)

Trip time (T_b) is defined as that time elapsed between burnout at earth and arrival at the moon. Each approach requires separate analysis for launch-on-time consideration. Holding the trip time constant has all the advantages consistent with guidance and tracking coordination, predicting crew necessities, return trip and reentry conditions, etc. However, it will be pointed out that fixed trip time carries severe late launch velocity penalties for the high ϕ inclinations. This situation may be relieved if a certain amount of trip time tolerance is introduced. A discussion of the above mentioned approaches follows.

1.5.2.1 Fixed Trip Time

Specification of constant trip time and the assumption that the moon is moving with constant angular rate about the earth implies a fixed relationship between the position of the moon at the time of

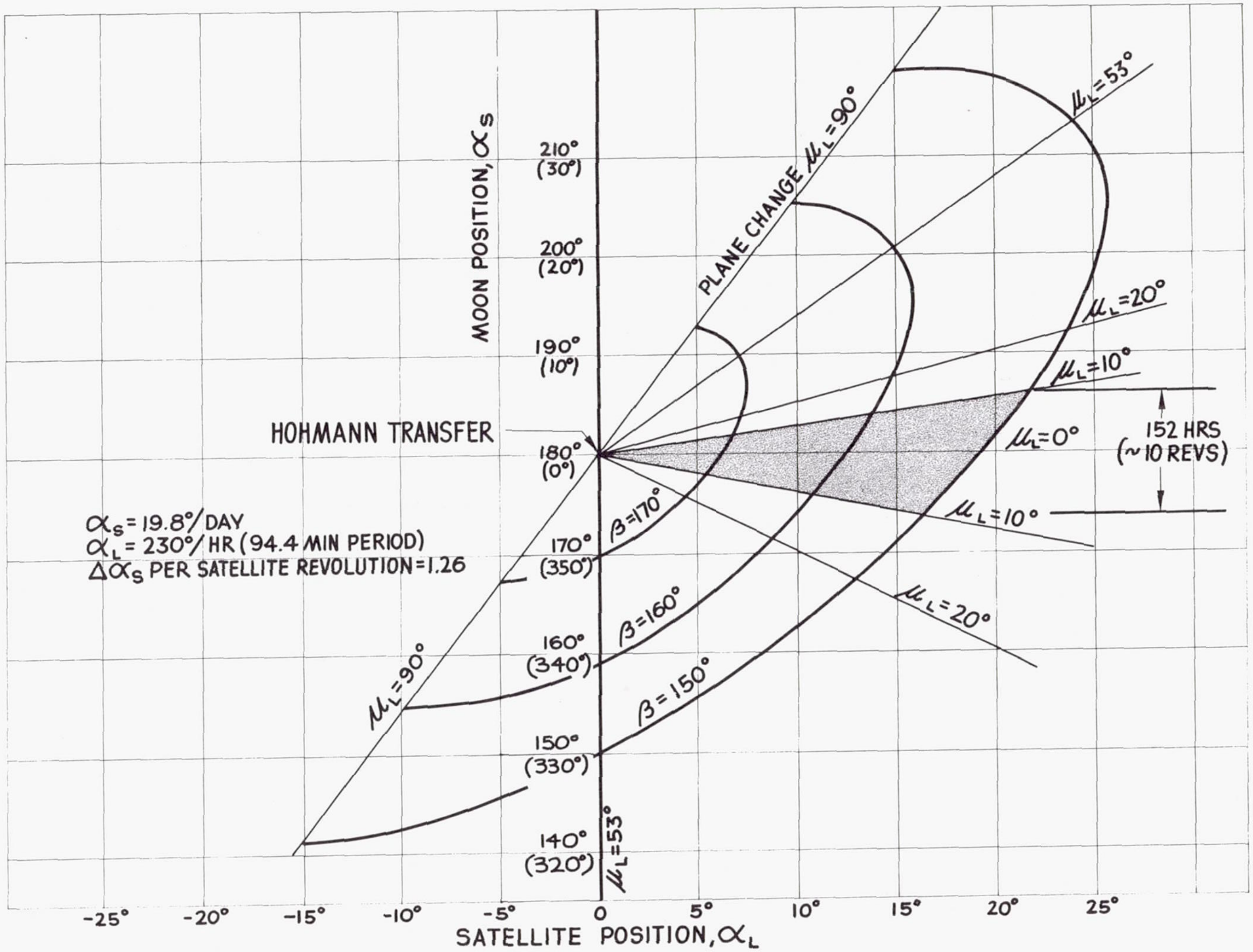


FIGURE 19 ENLARGED PARTIAL DISPLAY OF GEOMETRIC PARAMETERS, $\phi = 53°$

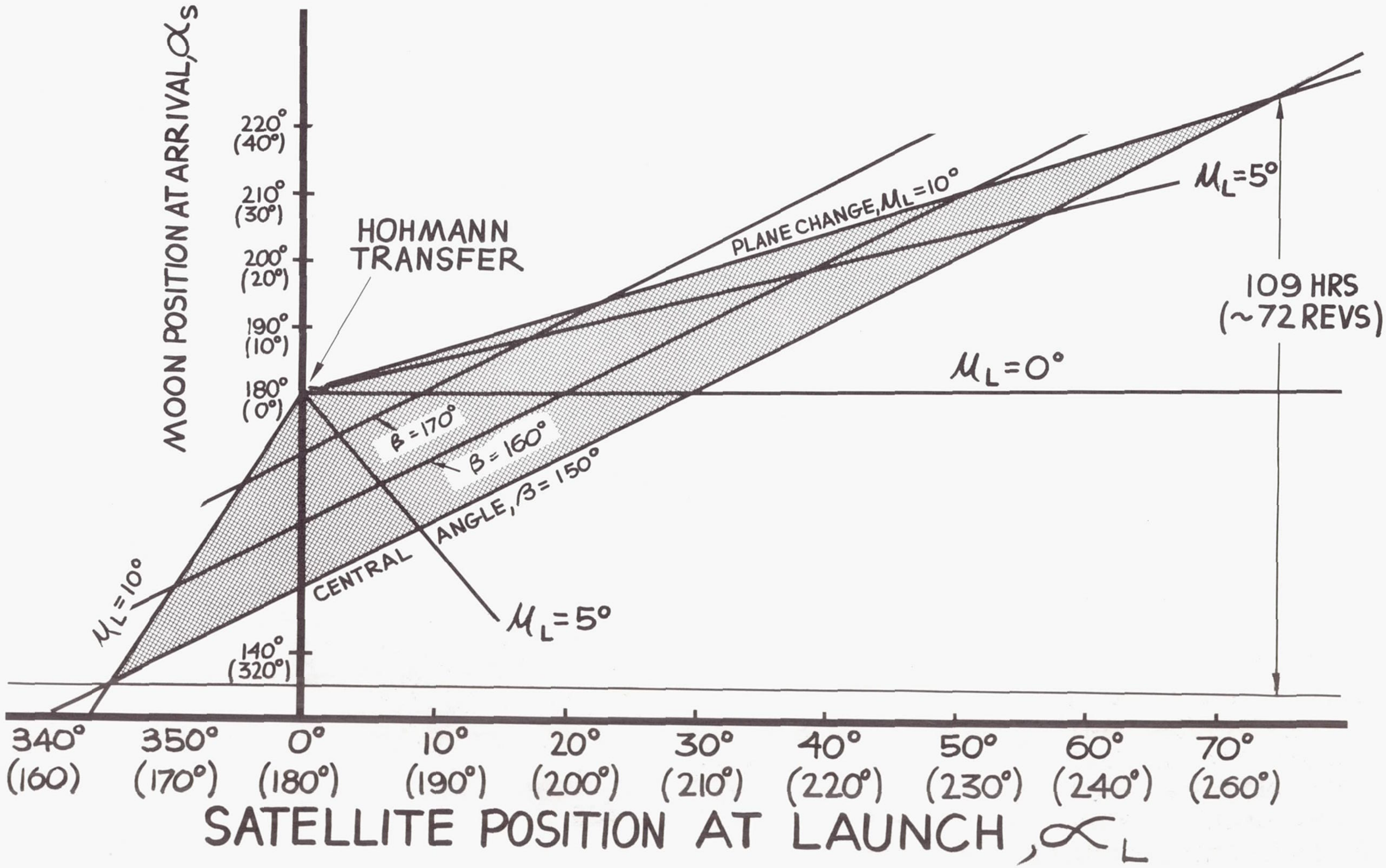


FIGURE 20 ENLARGED PARTIAL DISPLAY OF GEOMETRIC PARAMETERS, Φ = 7°

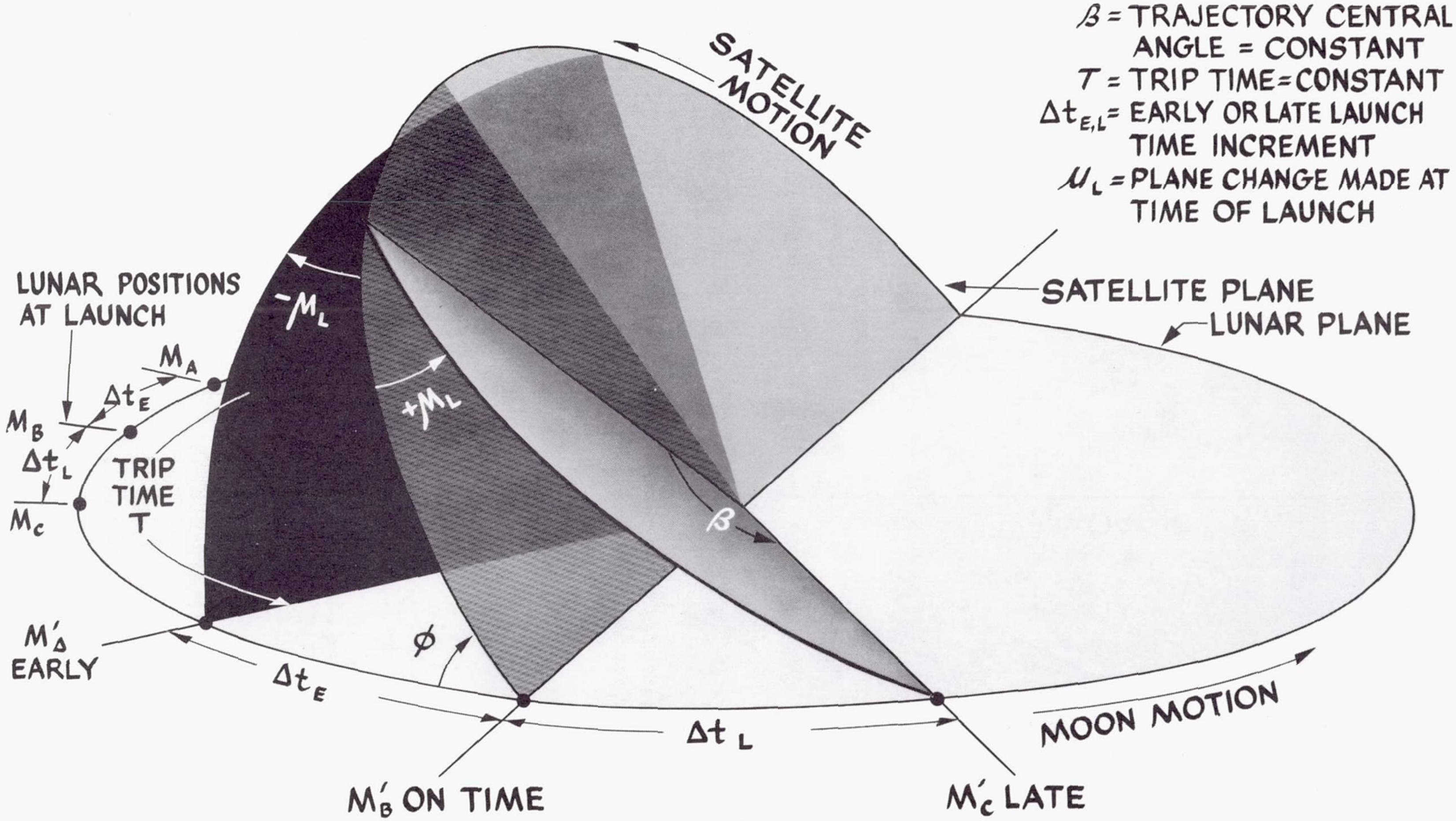


FIGURE 21 FIXED TRIP TIME TRAJECTORY DISPLAY

launch and its later position at the time of arrival. This is described in Figure 21. For this reason the arrival geometry can be used to describe launch conditions. Since constant trip time implies constant β, compensation for late (or early) launch must be made by a plane change adjustment μ_L. Figures 19 and 20 show the plane change and central angle requirements as a function of the positions of the satellite and moon. The feasible situations within the arbitrarily chosen constraints occur each time the lunar position is in the vicinity of the node line (α_s = 180° or 0°)

The time it takes the moon to pass through the escape window is a function of the relative motion of the moon and the OLF-lunar plane node line. If an average rate is used for node line precession as previously indicated, it is possible to calculate the time of passage of the moon beyond the line of nodes (late launch) for various values of ϕ. These values of late launch time for various ϕ's (10°, 20°, 30° and 50°) are plotted in Figures 22 through 25. These charts are a display similar to Figures 19 and 20 with the following exceptions. The position of the moon beyond the node line ($\alpha_s \geq 180°$) is shown on the vertical axis and this has been converted to time using the α_s rate. Since this is portrayal of conditions after the moon crosses the node line, it is a measure of the lateness of launch. Thus for a given fixed central angle β, the plane change requirement can be determined as a function of time. The velocity buildup as a function of lateness or μ_L is obtained from Figure 18. Using this approach with Figures 22 through 25, a correlation between ϕ, ΔV, and lateness of launch for a fixed trip time was obtained and is shown in Figures 26 through 31. A more precise determination of window size must account for non-linear motion of the OLF-Lunar planes node line.

For data presentation, a nominal point for reference should be selected. In the selection of this point, an attempt should be made to pick a realistic, optimum transfer trajectory. A nominal value for (170°) was selected which is large enough to be in the minimal portion of the vs V_{BO} plot (Figure 17), yet not in the excessive trip time portion of the t vs V_{BO} plot (Figure 17). A zero nominal plane change angle was also selected (since this is an optimum figure). An arrival altitude at the moon of 50 nautical miles is also used for reference. Such a nominal trajectory would have the following energy characteristics:

(a) $\beta = 170°$; $V_{BO} = 35{,}110$ fps

(b) $\mu_L = 0°$; $\Delta V_{ADD} - 0$ fps

(c) Arrival velocity at the 50 nautical mile lunar altitude = 8530 fps

(d) A lunar retrothrust increment of 8530 - 5380 = 3150 fps must be supplied to achieve lunar orbit.

The total nominal velocity increment which the reference trajectory requires for a lunar orbit mission is obtained by adding the

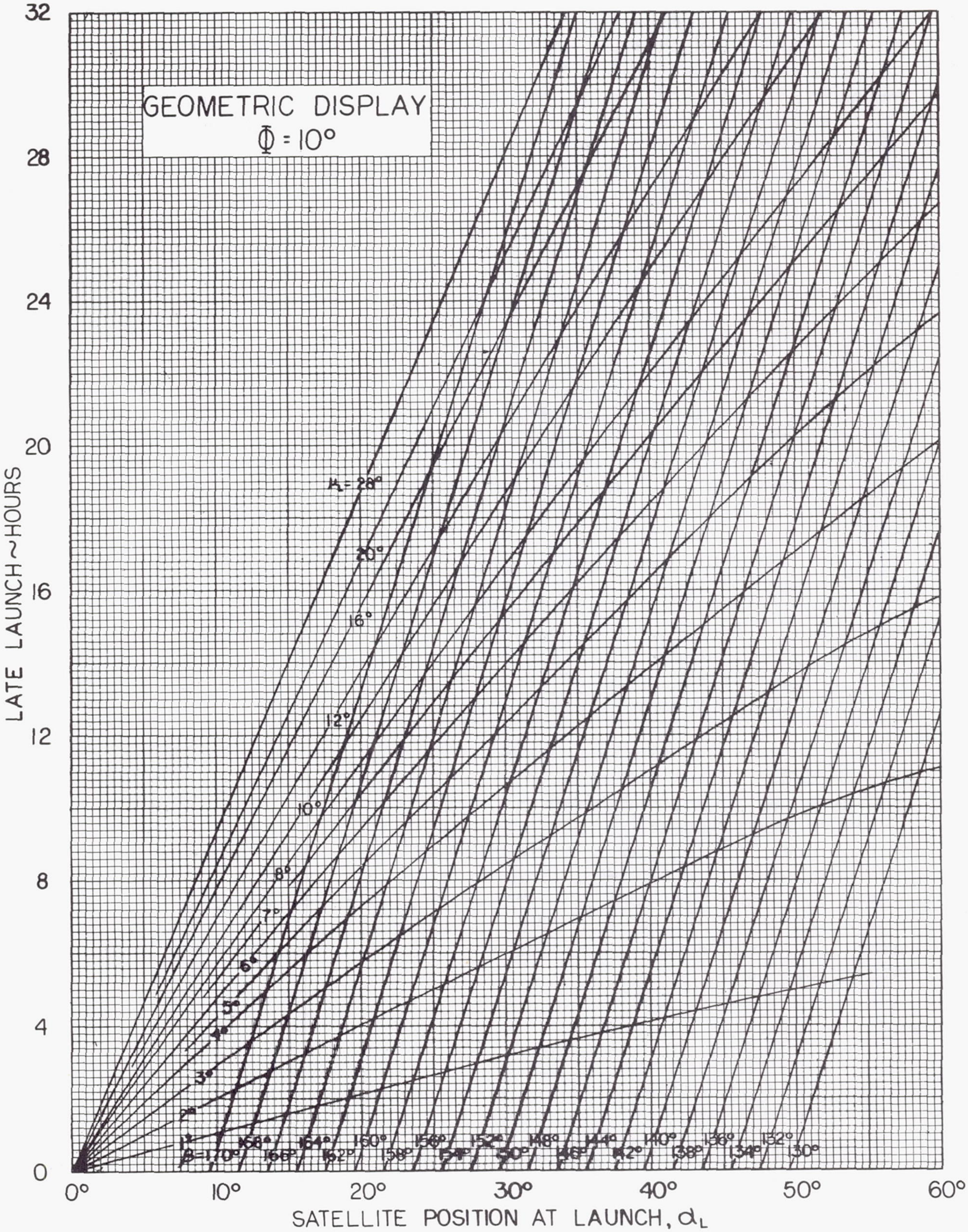


FIGURE 22 LATE LAUNCH GEOMETRY DISPLAY $\phi = 10°$

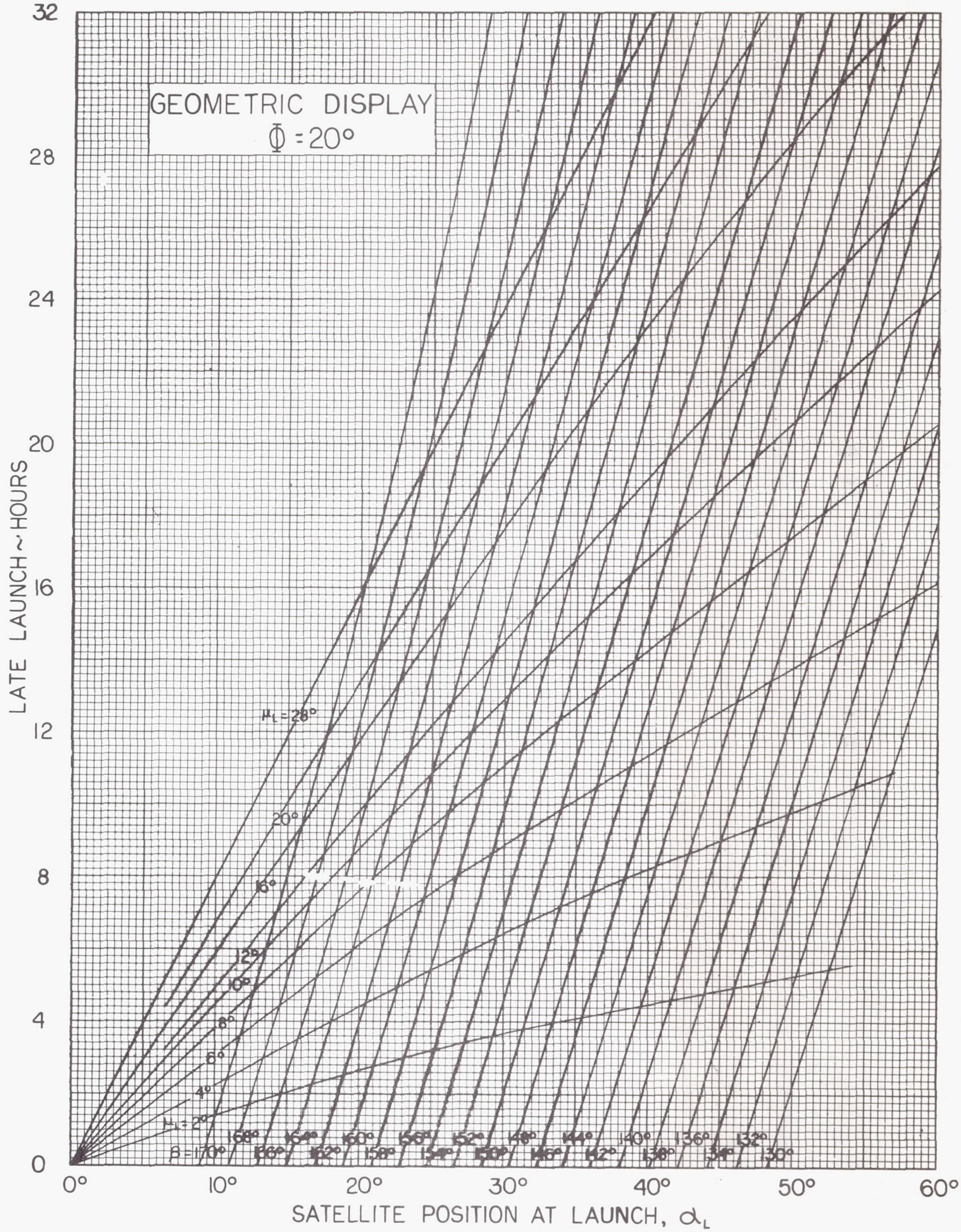


FIGURE 23 LATE LAUNCH GEOMETRY DISPLAY ϕ = 20°

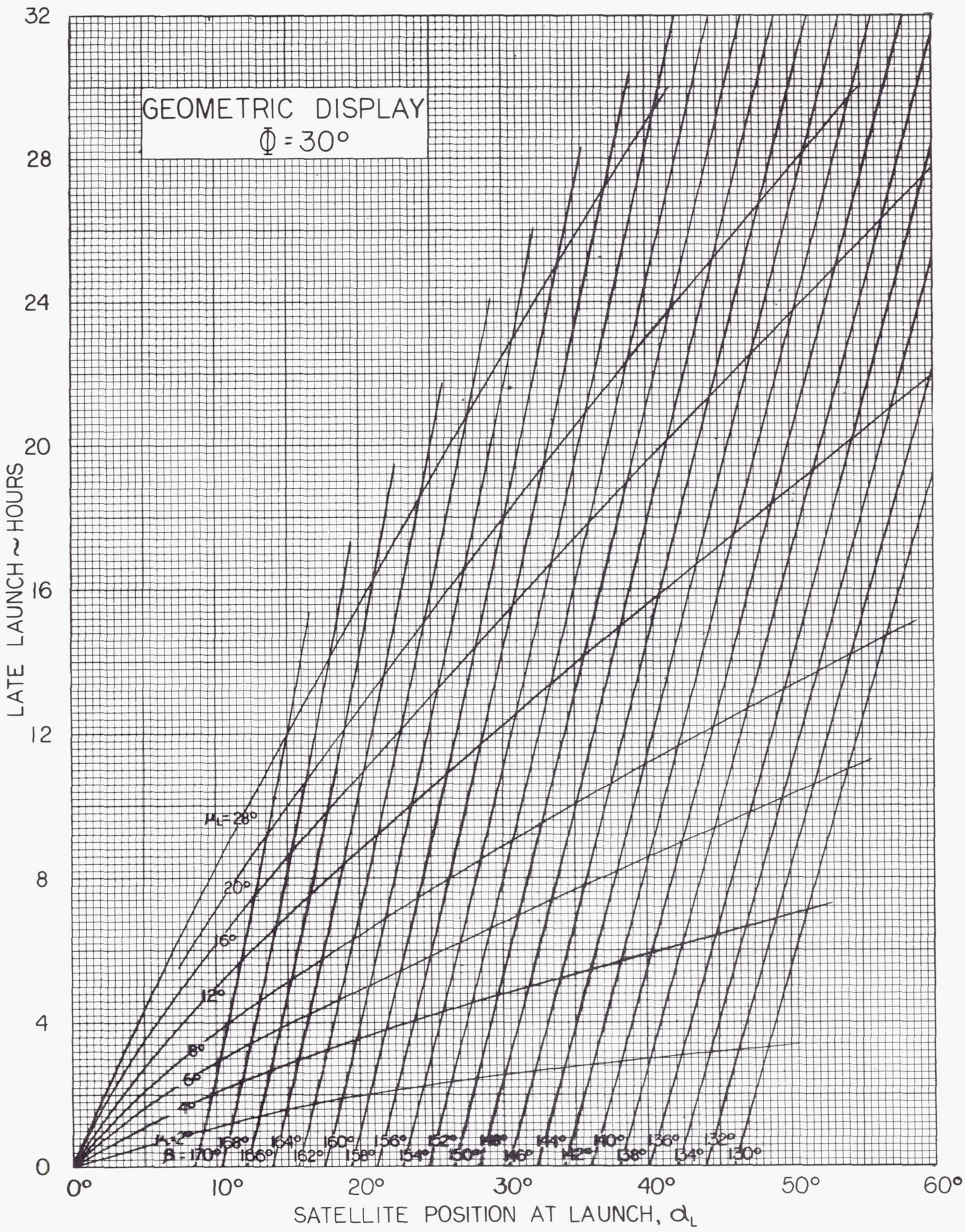


FIGURE 24 LATE LAUNCH GEOMETRY DISPLAY, ϕ = 30°

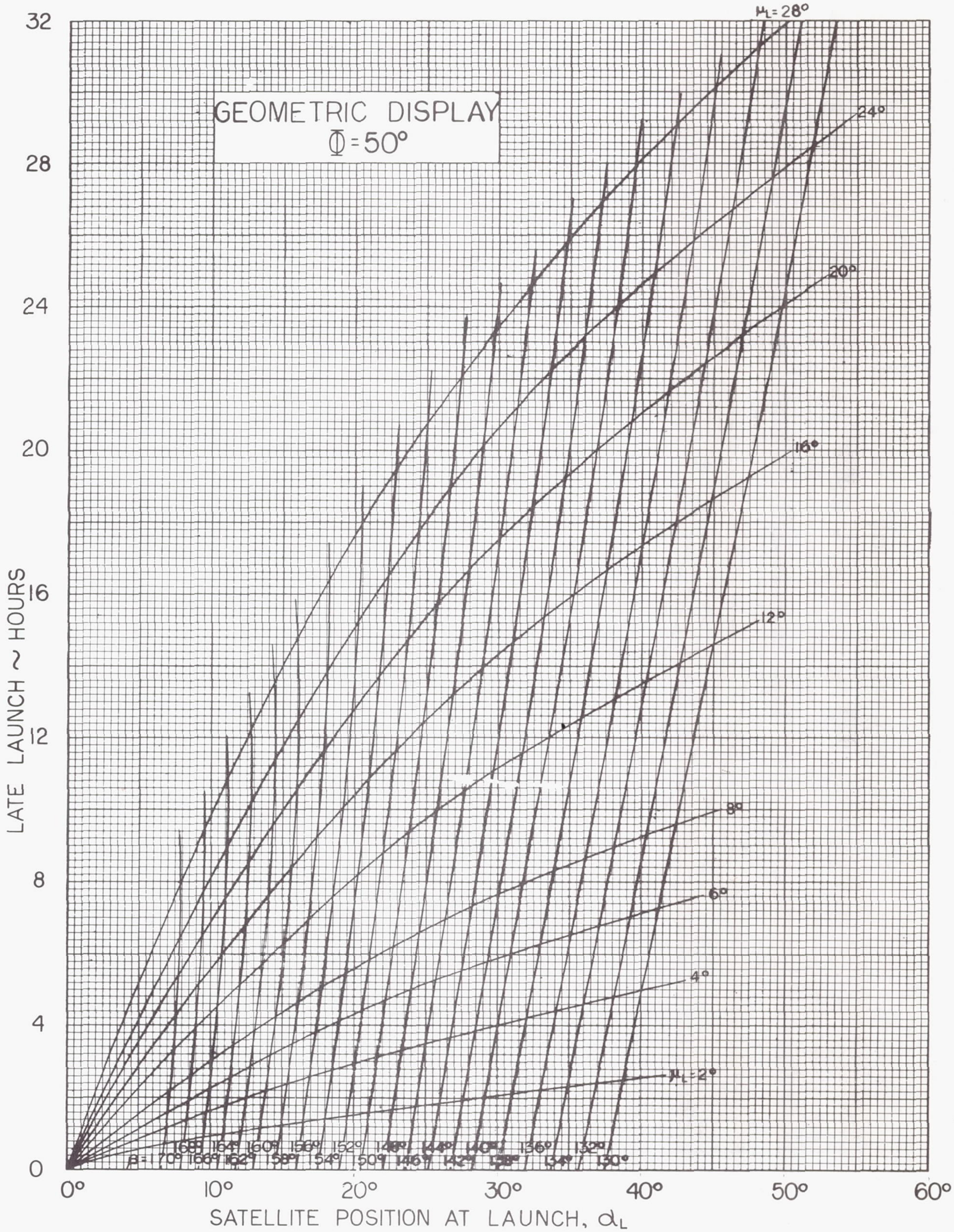


FIGURE 25 LATE LAUNCH GEOMETRY DISPLAY, $\phi = 50°$

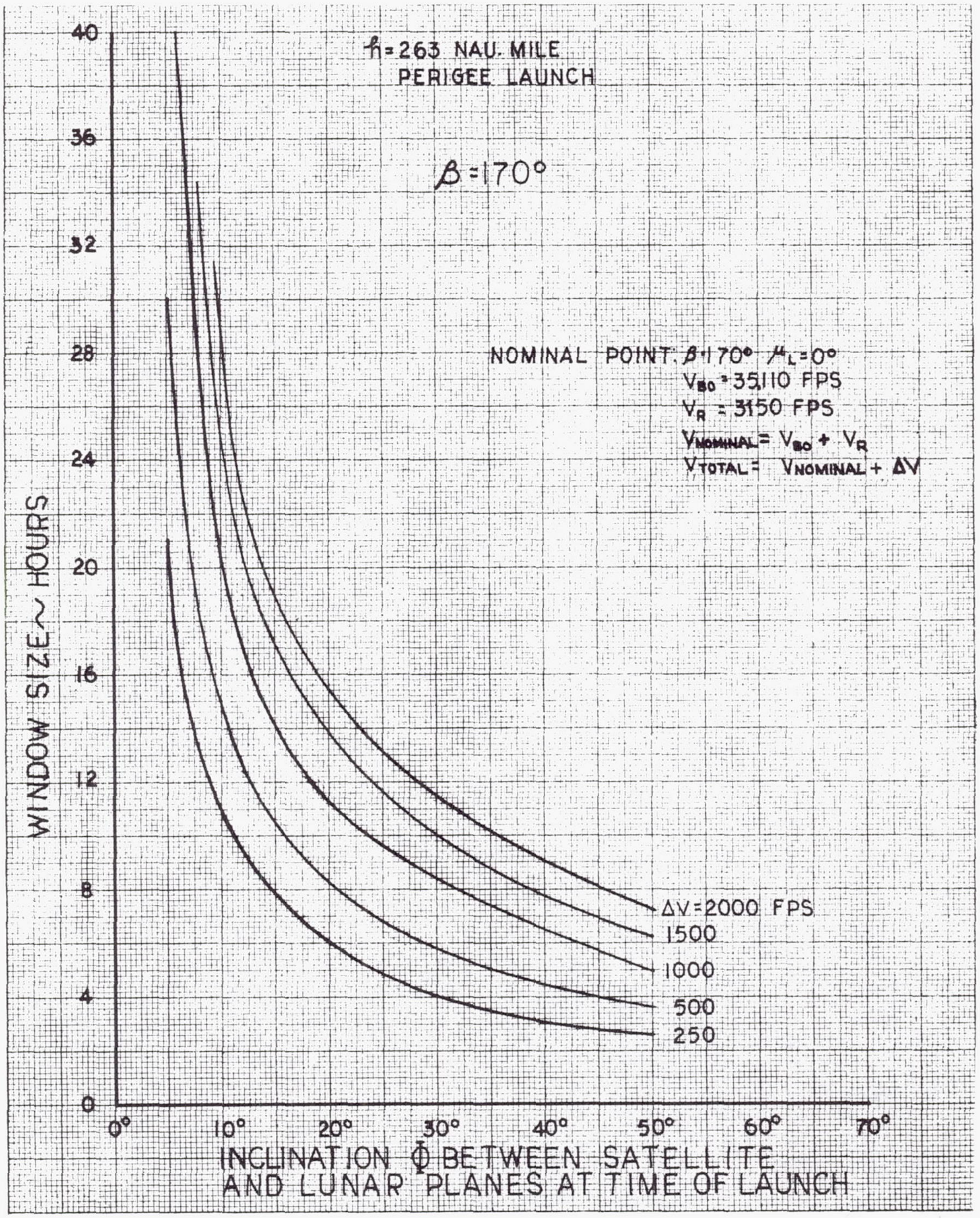


FIGURE 26 FIXED TRIP TIME WINDOW SIZE, β = 170°

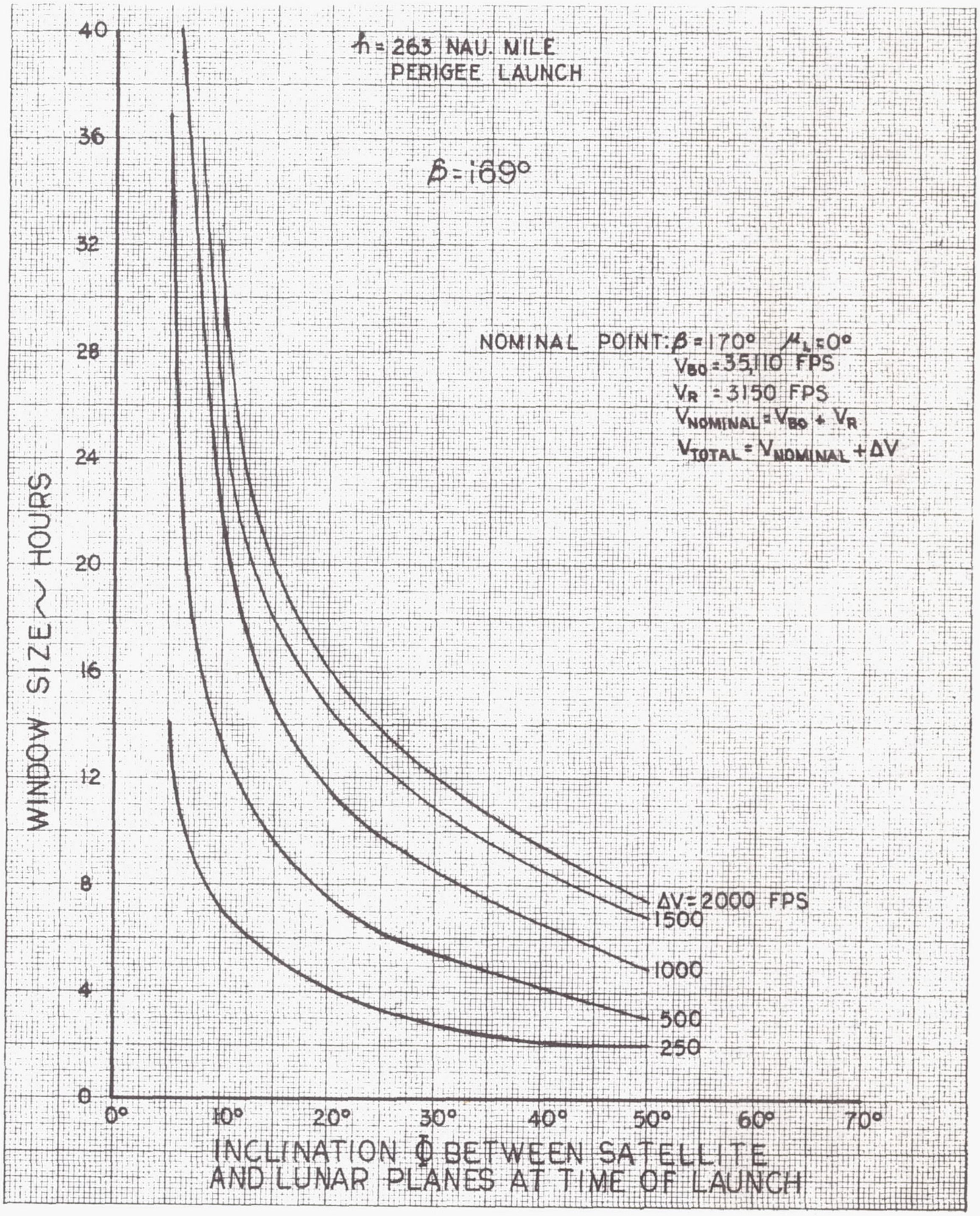


FIGURE 27 FIXED TIME WINDOW SIZE, β = 169°

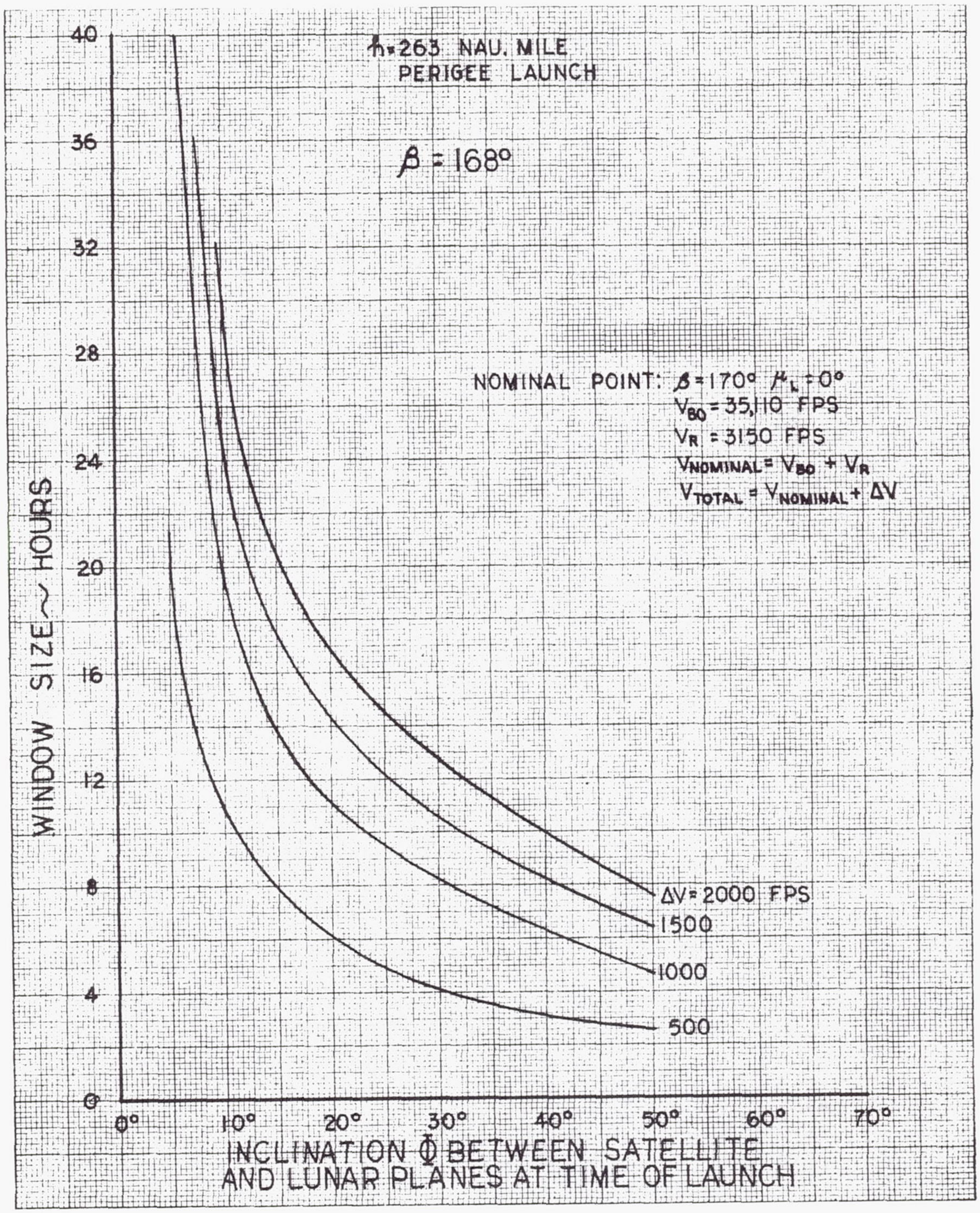


FIGURE 28 FIXED TRIP TIME WINDOW SIZE, $\beta = 168°$

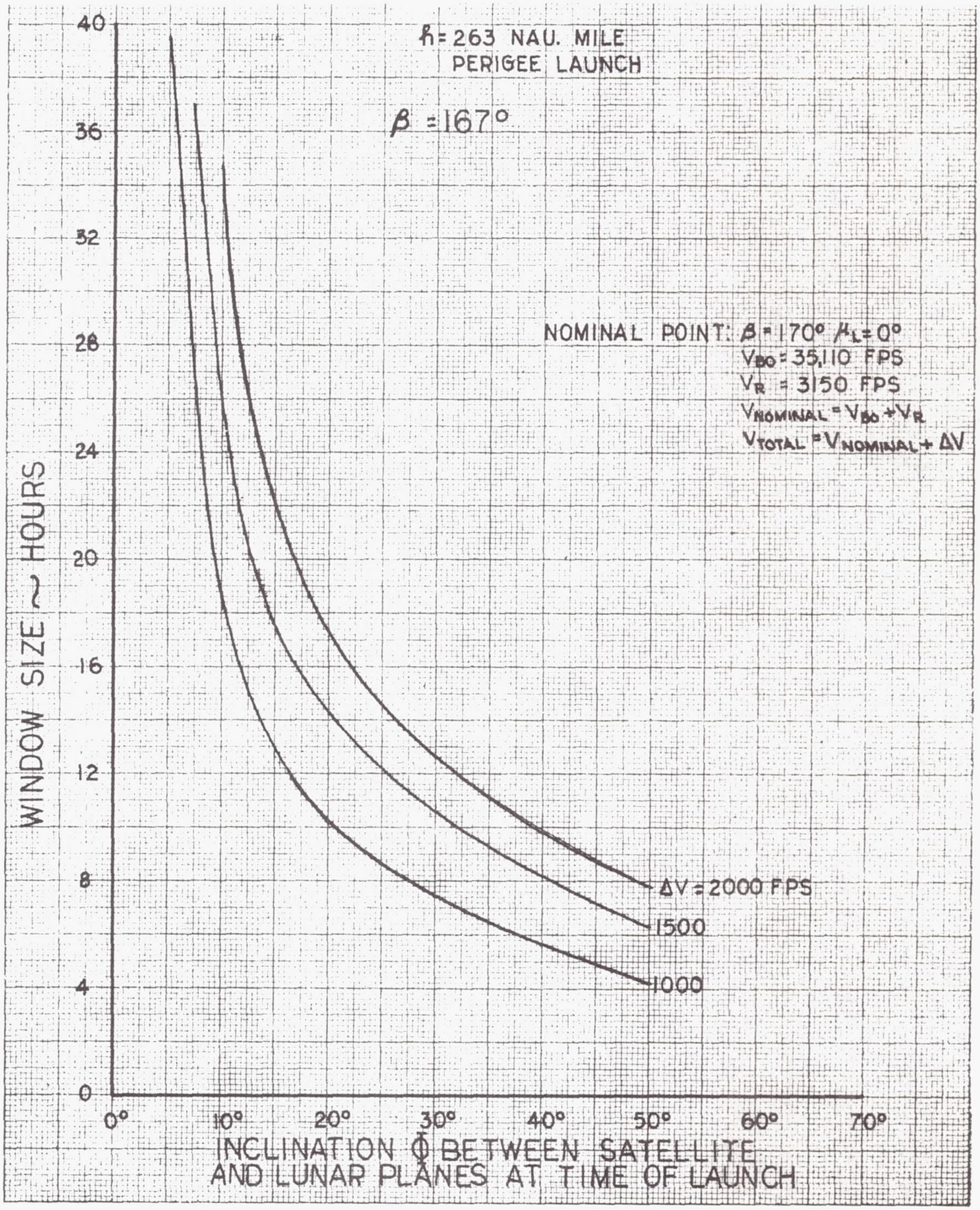


FIGURE 29 FIXED TRIP TIME WINDOW SIZE, β = 167°

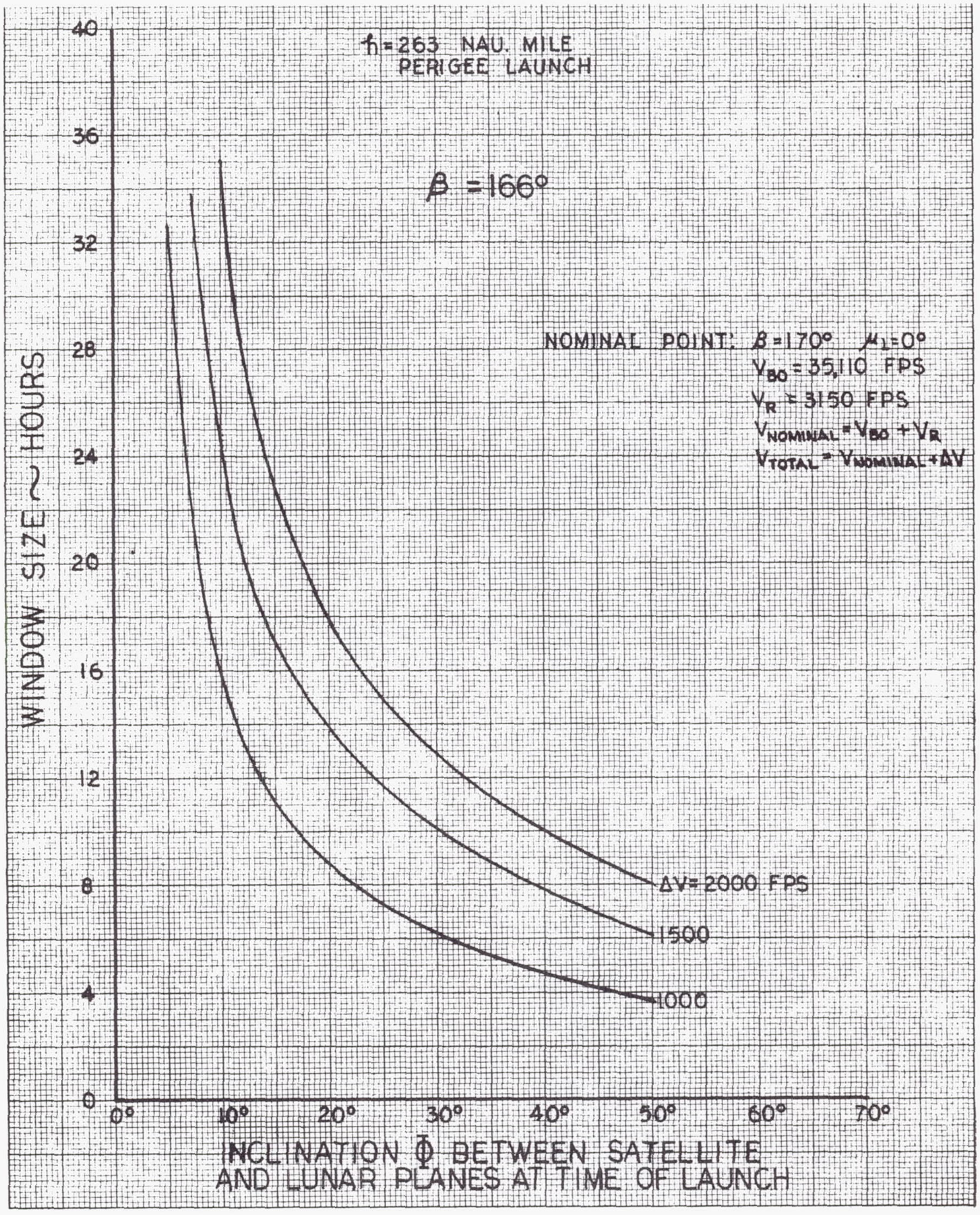


FIGURE 30 FIXED TRIP TIME WINDOW SIZE, $\beta = 166°$

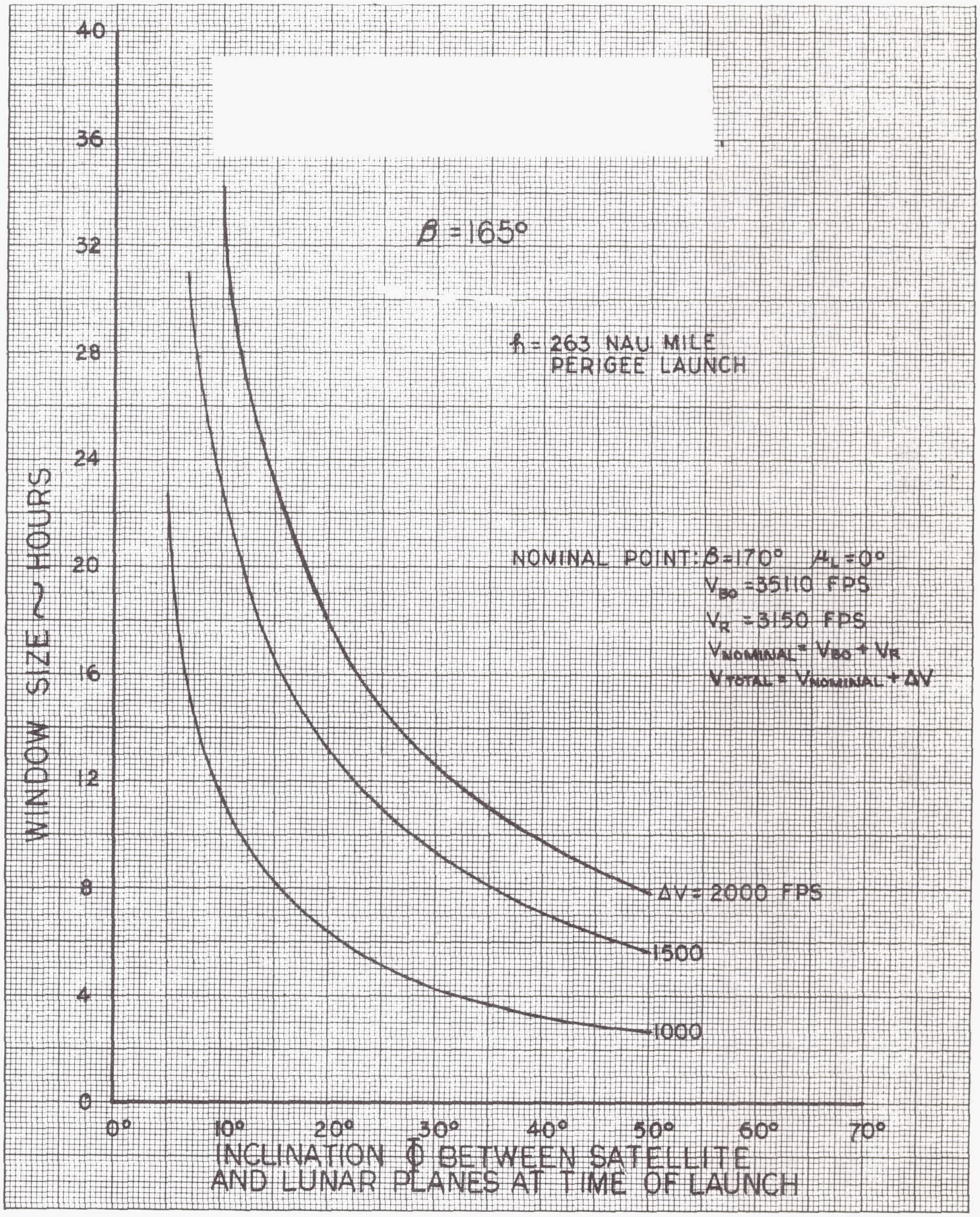


FIGURE 31 FIXED TRIP TIME WINDOW SIZE, β = 165°

various components -

$$\Delta V_{nominal} = (35{,}110 + 0 + 3150) = 38{,}260 \text{ fps}$$

Actually, since the OLF at the specified altitude (h = 263 nautical miles) already has a circular velocity of 24,999 fps, the total additional energy to escape earth orbit and capture into lunar orbit is (38,260 - 24,999) = 13,261 fps for the nominal case mission. The additional energy packages, ΔV_{ADD}, which are used to parameterize the window analysis are increments in addition to the nominal amount of 38,260 fps and represent fuel carried for deviations from the nominal. Figures 26 through 31 show that for a given ΔV_{ADD}, the window size decreases with decreasing β. This factor is important in the overall mission analysis to emphasize the effect of any changes of the escape condition on the lunar end of the mission with the resulting effect on escape window size.

To illustrate the use of Figures 17 through 31 in mission analysis and the determination of the escape window size, the following example is presented.

PROBLEM - To determine escape window size for $\beta = 168°$, $\phi = 10°$, $\Delta V_{ADD} = 500$ fps

(a) Total allowable velocity $= V_{nominal} + \Delta V_{ADD}$
$= 38{,}260 + 500$
$= 38{,}760$

(b) Determine in-plane velocity requirement for $\beta = 168°$ from Figure 17

$$V_{BO} = 35{,}170$$

(c) Determine arrival velocity at the 50 nautical mile moon altitude from Figure 17 for the $V_{BO} = 35{,}170$

$$V_{arr_M} = 8{,}800$$

(d) The retrothrust requirement to enter circular orbit at moon.

$$(8800 - 5380) = 3{,}420$$

(e) The sum of steps b and d

$$(35{,}170 + 3420) = 38{,}590$$

(f) The amount of velocity available for plane change at launch, while still remaining within the allowable excess energy package, is step a minus step e

$$(38{,}760 - 38{,}590) = 170 \text{ ft/sec.}$$

(g) Figure 18 shows that a 3.6° plane change can be made with 170 ft/sec.

(h) Figure 22($\phi = 10°$) shows that the $\beta = 168°$ curve intersects the 3.6° μ_L curve 5.3 hours after optimum time. Since this amount of time is also available for early launch, the total window size is 2 x 5.3 = 10.6 hours. A check of Figure 28 shows this point.

(i) Apply the adjustment factor for non-linear node line precession rate (discussed in paragraph 1.6). Figure 43 shows for

$\phi \approx 10°$ and $\eta \approx 27°$ (assume year 1967), the adjustment factor is 0.92:
$\therefore$ the window is $0.92 \times 10.6 \approx 9.75$ hrs.

The above problem has used a nominal mission as a reference to illustrate the general case. These nominal values are dependent on the type of trajectory analysis techniques and computer programs available. This fact is pointed out since <u>any</u> nominal mission can be used and the values depend on the assumptions made by the analyst.

The above problem has used a nominal mission as a reference to illustrate the general case. These nominal values are dependent on the type of trajectory analysis techniques and computer programs available. This fact is pointed out since <u>any</u> nominal mission can be used and the values depend on the assumptions made by the analyst.

Since the summary curves (26 - 31) do not go below $\phi \approx 5°$, Figure 32 was included to describe the additional velocity increment as a function of lateness of launch for constant ϕ values below 5°. Window size, which includes early launch time as well, is determined by doubling the time for a given velocity increment. This curve is particularly pertinent because it represents a time history of velocity buildup for ϕ values typical of first opportunity windows which will exist in the late 60's. Since maximum window size will occur near minimum ϕ, orbital launch missions will utilize this fact by synchronizing the launch window near minimum ϕ. Minimum attainable ϕ equals the latitude of the launch site (AMR latitude $\approx 28.5°$) minus the inclination of the lunar plane to the equator. Figure 32 does not include values below $\phi \approx 3°$ because node line rate non-linearities become extremely important in this range and their effect has not been completely defined.

1.5.2.2 Variable Trip Time

While the fixed trip time approach involves holding the transfer trajectory central angle constant and making a plane change to allow for late or early launch, another possibility for adjusting for lateness exists which would not involve any plane change, requiring instead a reduction in the central angle. This is described in Figure 33. Although the picture shows the moon at arrival fixed in space, it should be remembered that the transfer plane is moving in space (coplanar with satellite plane) so that the moon at arrival is moving at an angular rate equal to the precessional motion of the satellite plane. A reduction in central angle results in a faster transfer. This is a more optimum procedure because an adjustment in central angle for a given increment of tardiness involves less additional energy than a plane change. However, the variable trip time technique which is considered here actually involves a combination of plane change and central angle change. That is, for a given position of the moon at the time of launch, there exists a combination plane change and central angle which requires a minimum energy mission velocity. Also to be considered in the variable trip time approach is

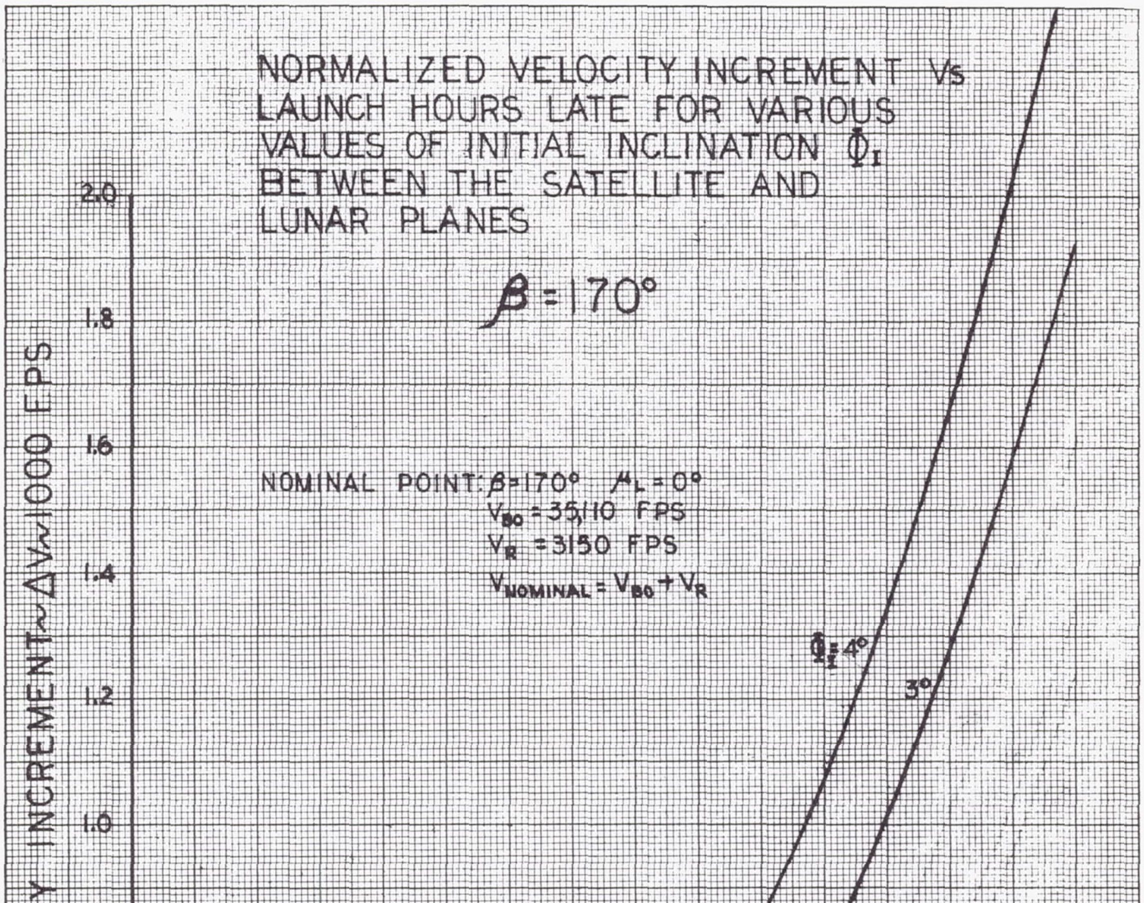


FIGURE 32 LATE LAUNCH VELOCITY PENALTY FOR SMALL ϕ's

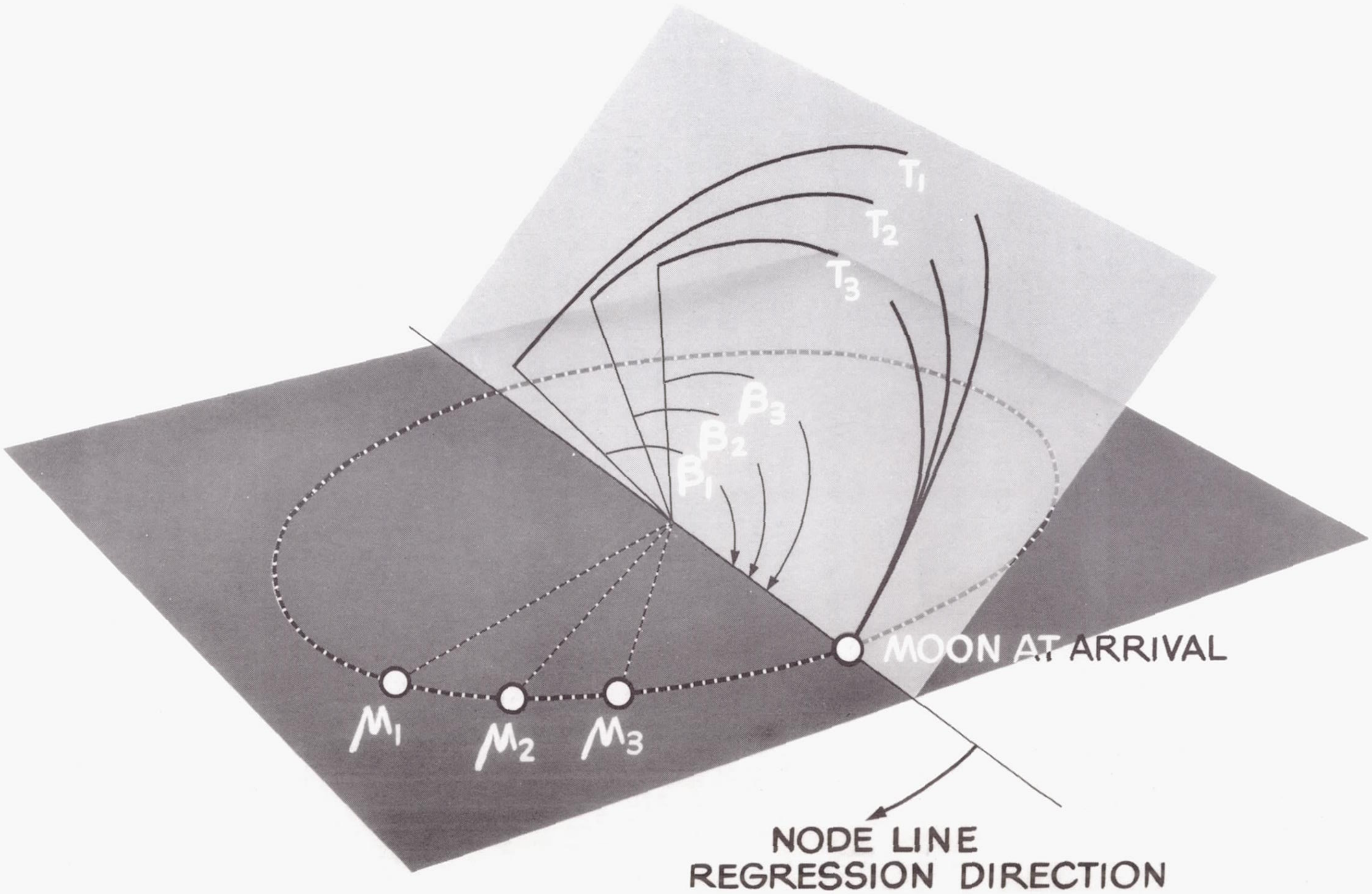


FIGURE 33 VARIABLE TRIP TIME TRAJECTORY DISPLAY

the fact that as central angle decreases, inplane burnout velocity and arrival velocity at the moon increase. This was described in Figure 17 and the sample problem in the previous section. A retro-thrust increment is anticipated at the moon for landing or orbiting. This increment will increase as the central angle decreases and is accounted for when comparing the fixed and variable trip time approaches.

Figures 34 through 37 are geometry plots for the graphical determination of variable trip time window vs ϕ for various ΔV_{ADD}. The velocity increments have again been normalized about the β = 170°, μ_L = 0° nominal point. The μ_L lines were constructed by considering, for a given position of the moon at launch, all the possible combinations of β and μ_L which will cause lunar intercept. These are displayed along with the velocity requirement. The optimum line for a given ϕ is determined by drawing a line through the bottom of each iso-ΔV_{ADD} line.

The window size for a given ΔV_{ADD} is then determined by considering the total time which the moon at launch (shown on the vertical axis) is within a prescribed iso-velocity limit. For comparison purposes, consider the 500 fps velocity increment discussed in the fixed trip time section for a ϕ = 10°. Figure 34 shows that, as the moon approaches the line of nodes, the first opportunity for launch occurs when the moon is 74.3 hours from the node line. Finally, at 53.6 hours, the moon can no longer be reached with the given velocity allowance. Total time elapsed is 74.3 - 53.6 = 20.9 hours and this is the escape window for these conditions. Using Figures 34 through 37 in this fashion, the ΔV_{ADD}, the window size and ϕ parameterization was made and the results are shown in Figure 38A. Figure 38B shows a summary of the optimum late launch transfer trajectories. The change in velocity and central angle for a given ϕ as lateness increases is illustrated.

1.5.3 Comparison of Techniques

A comparison of the results of the variable and fixed trip time approaches can be seen in Figures 39 through 41. The advantage which the variable trip time technique holds on the fixed trip time approach is clearly demonstrated. It should be emphasized that any lunar shot from orbit would most likely be synchronized to coincide with near minimum ϕ values. Only when the initial attempt fails and the satellite plane is committed to a given ϕ cycle, do the higher ϕ inclinations become pertinent. Values of ϕ for a typical series of launch opportunities were shown in Figure 4. The change in ϕ during waiting periods is apparent.

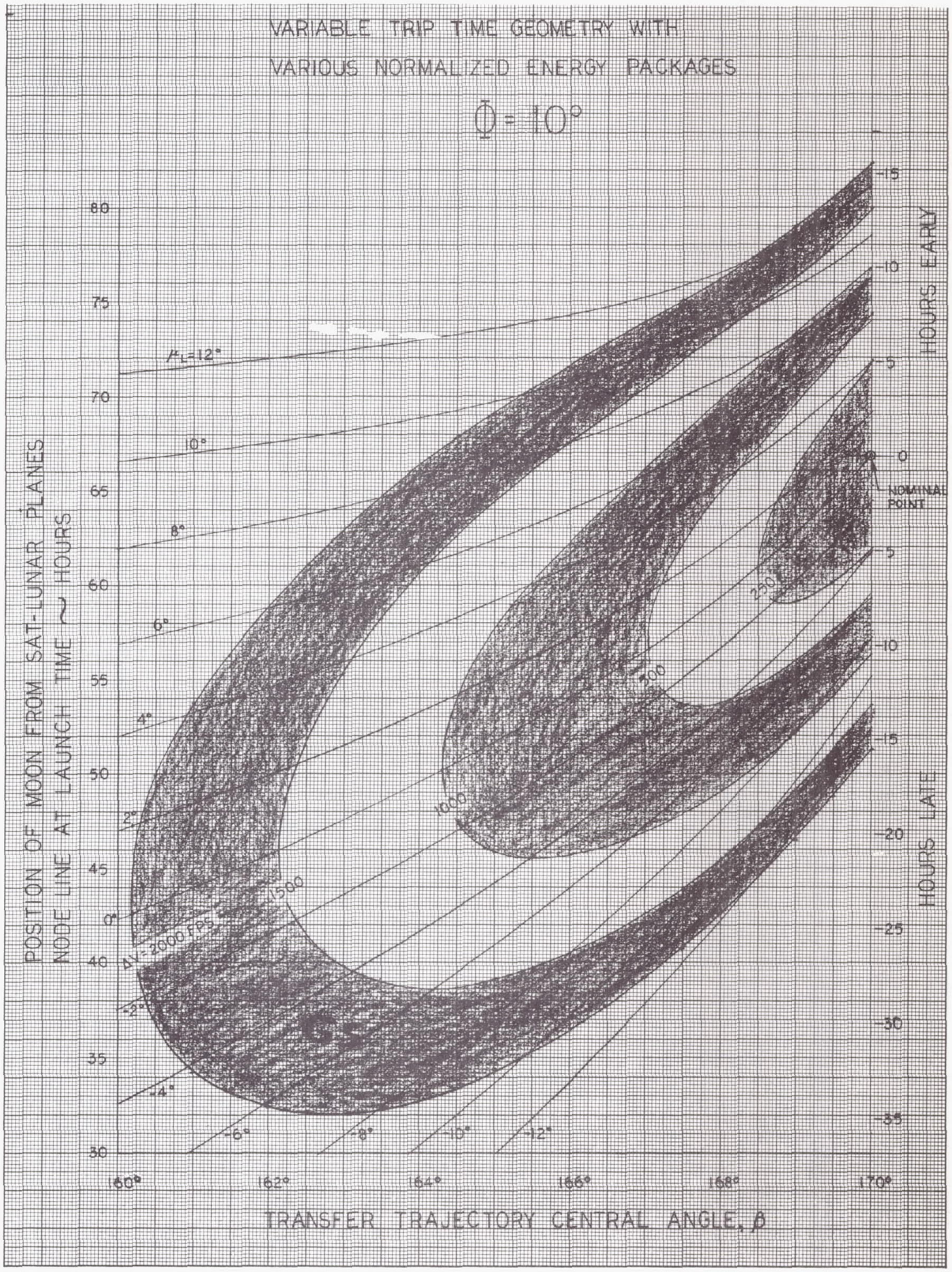


FIGURE 34 VARIABLE TRIP TIME GEOMETRY DISPLAYS $\phi = 10°$

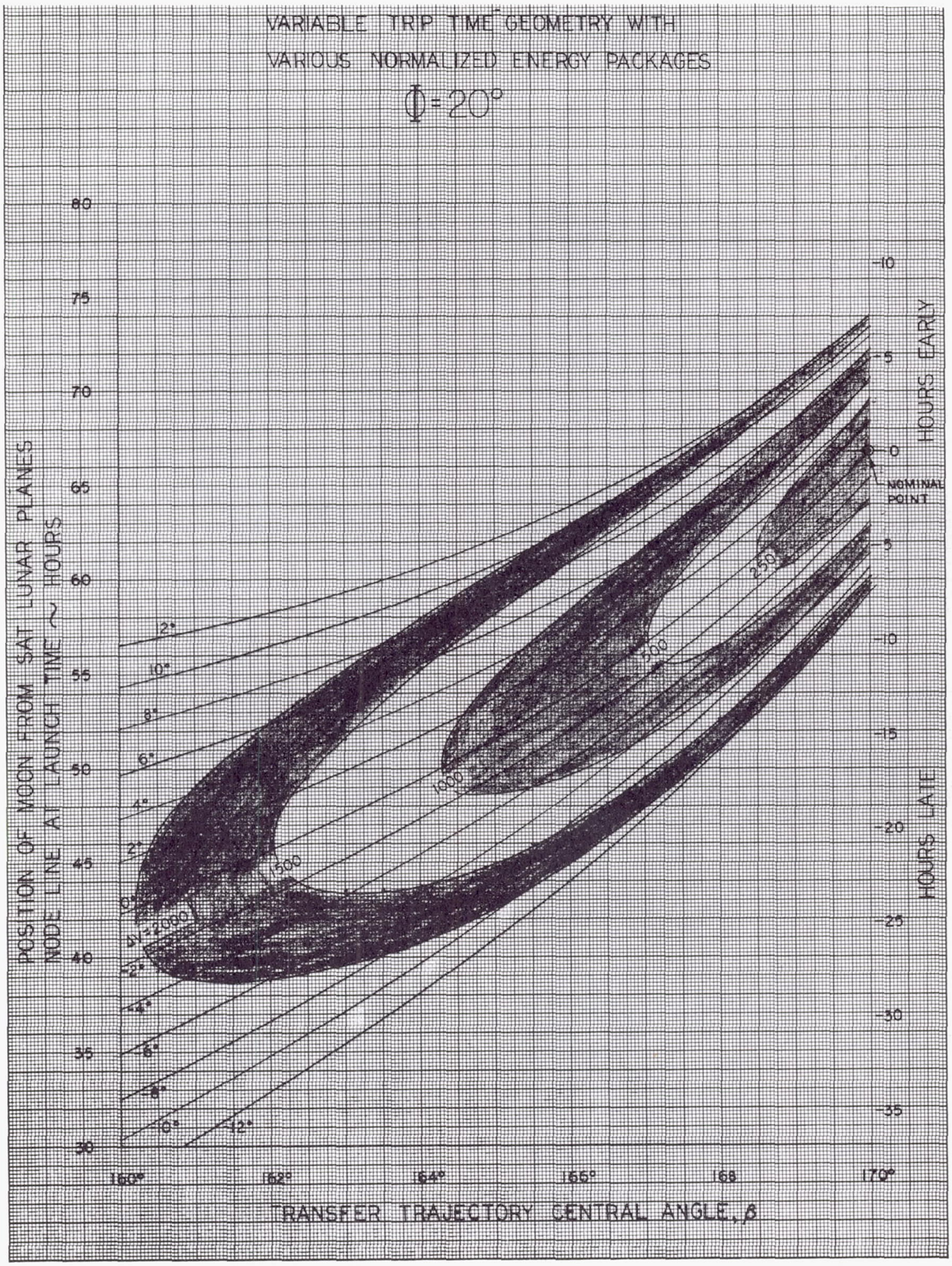


FIGURE 35 VARIABLE TRIP TIME GEOMETRY DISPLAYS $\phi = 20°$

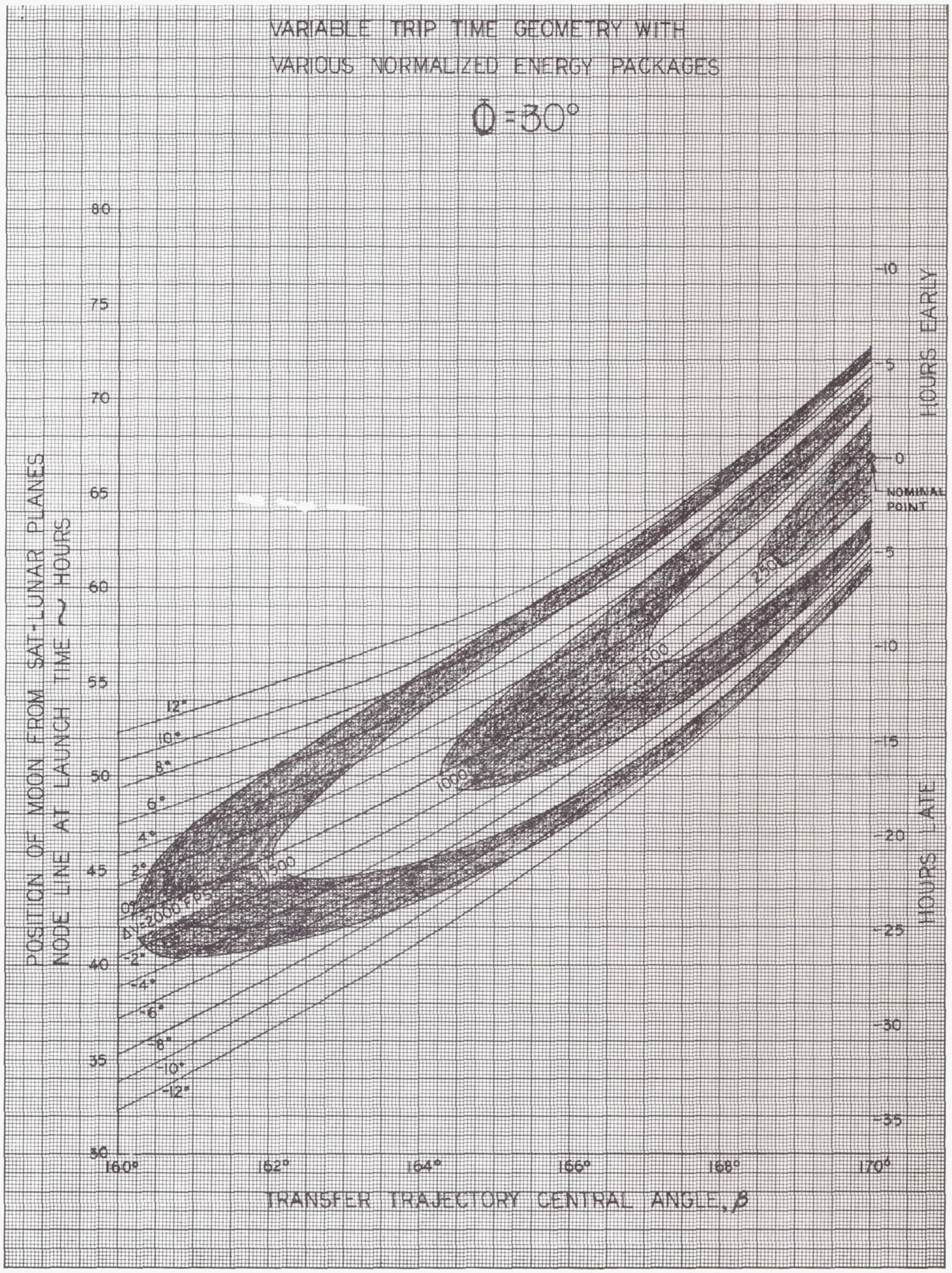


FIGURE 36 VARIABLE TRIP TIME GEOMETRY DISPLAYS $\phi = 30°$

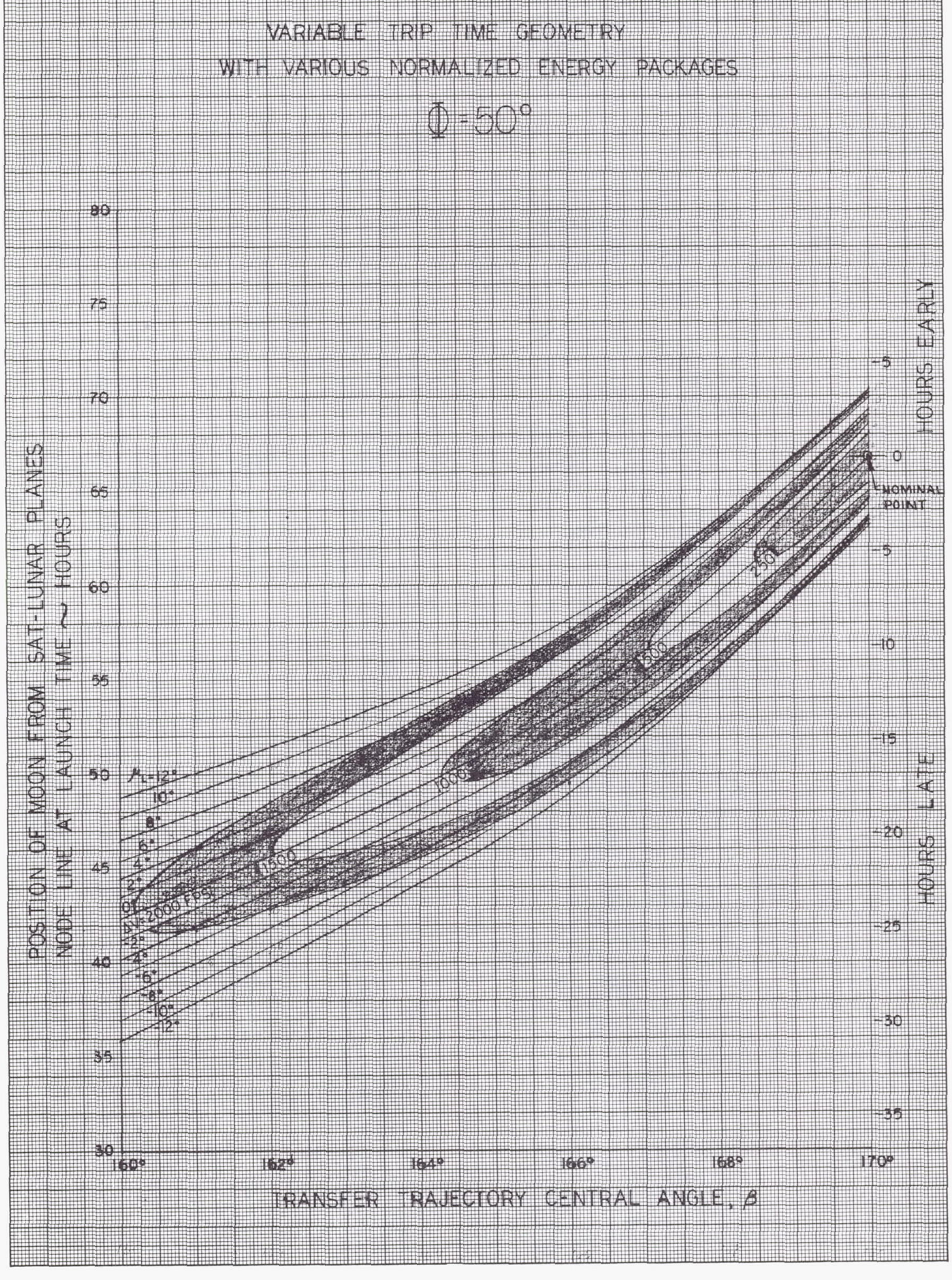


FIGURE 37 VARIABLE TRIP TIME GEOMETRY DISPLAYS $\phi = 50°$

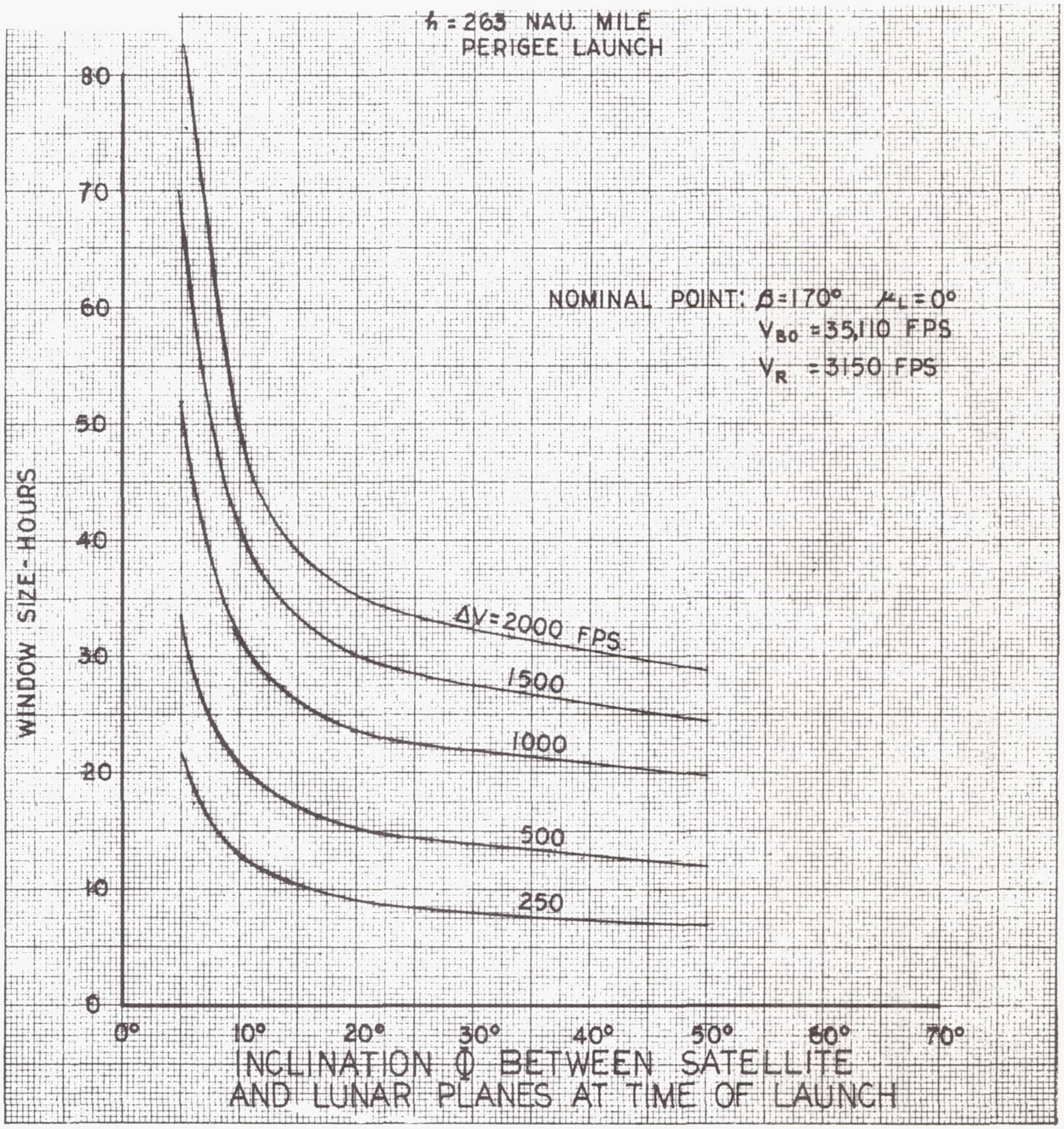


FIGURE 38A VARIABLE TRIP TIME ESCAPE WINDOW

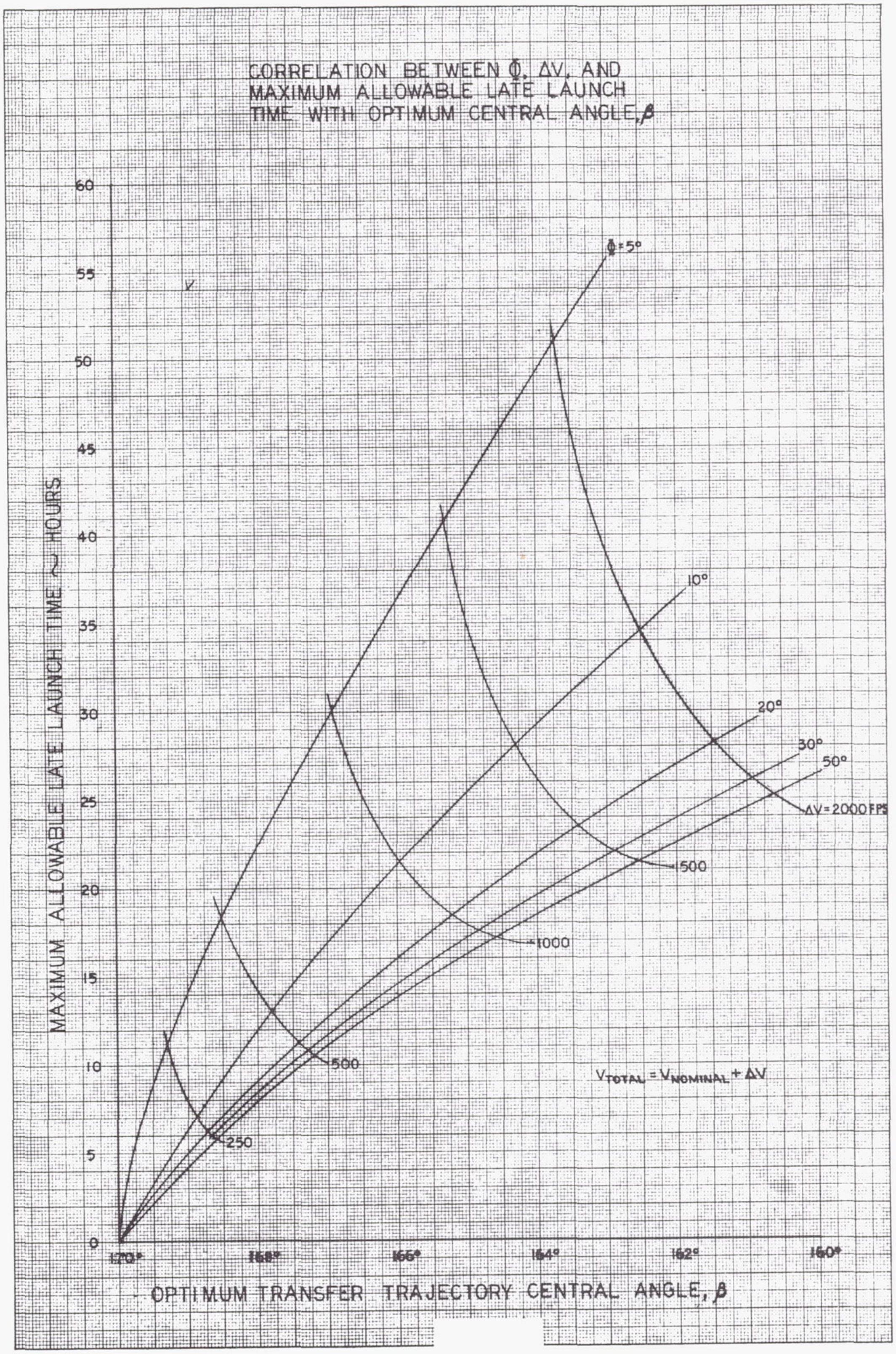


FIGURE 38B VARIABLE TRIP TIME LATE LAUNCH OPTIMIZATION SUMMARY

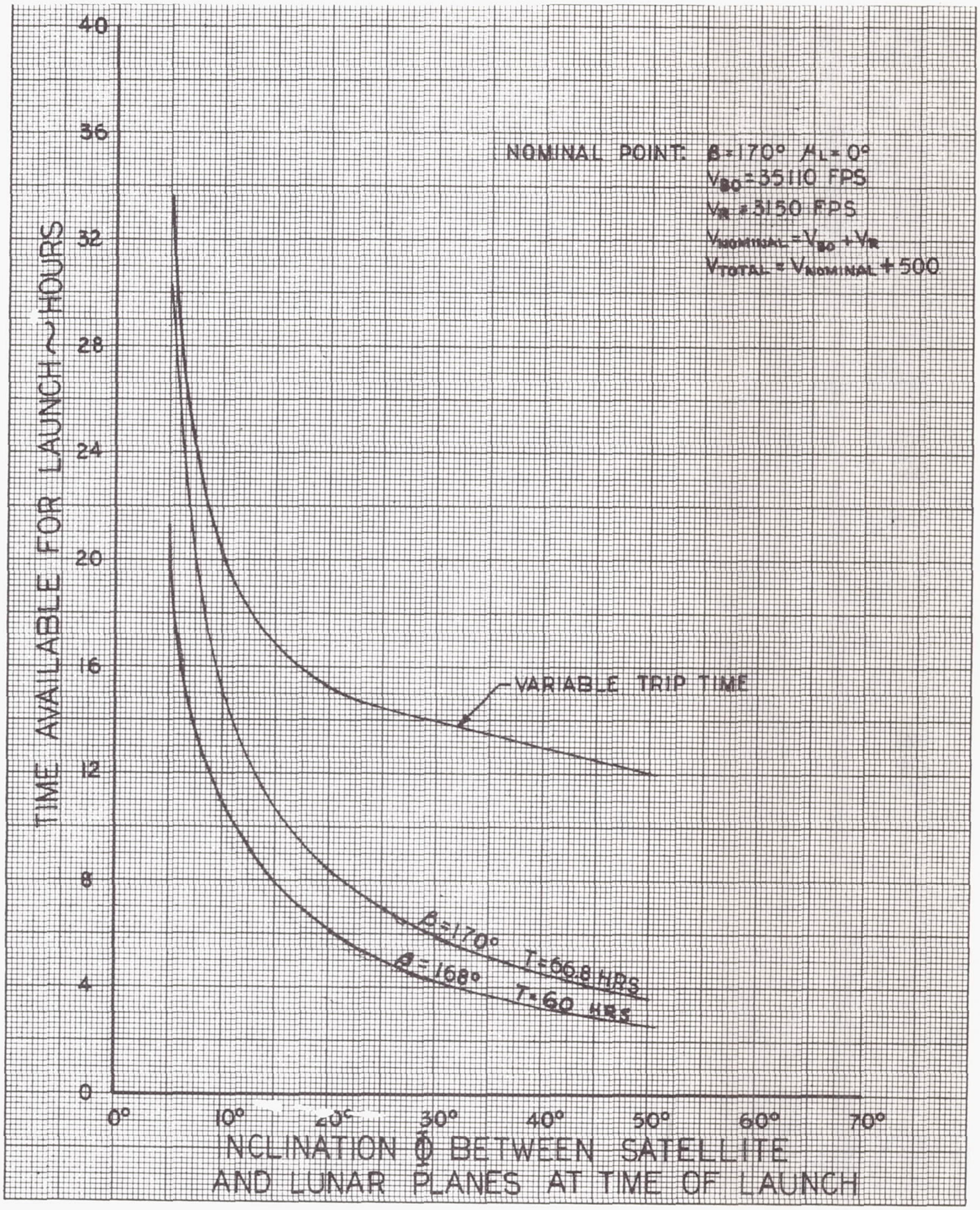


FIGURE 39 FIXED AND VARIABLE TRIP TIME ESCAPE WINDOW COMPARISON ΔV = 500 FPS

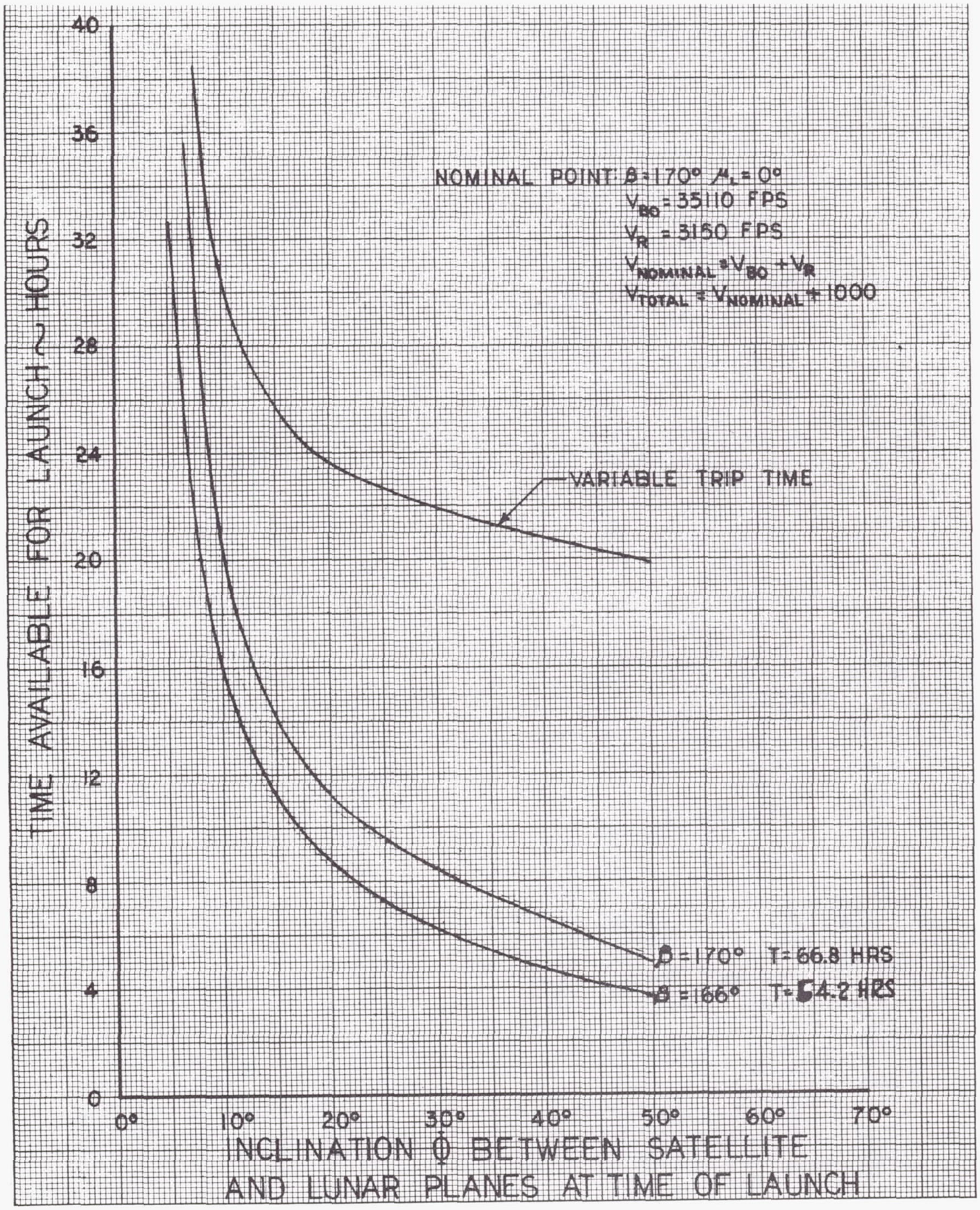


FIGURE 40 FIXED AND VARIABLE TRIP TIME ESCAPE WINDOW COMPARISON ΔV = 1000 FPS

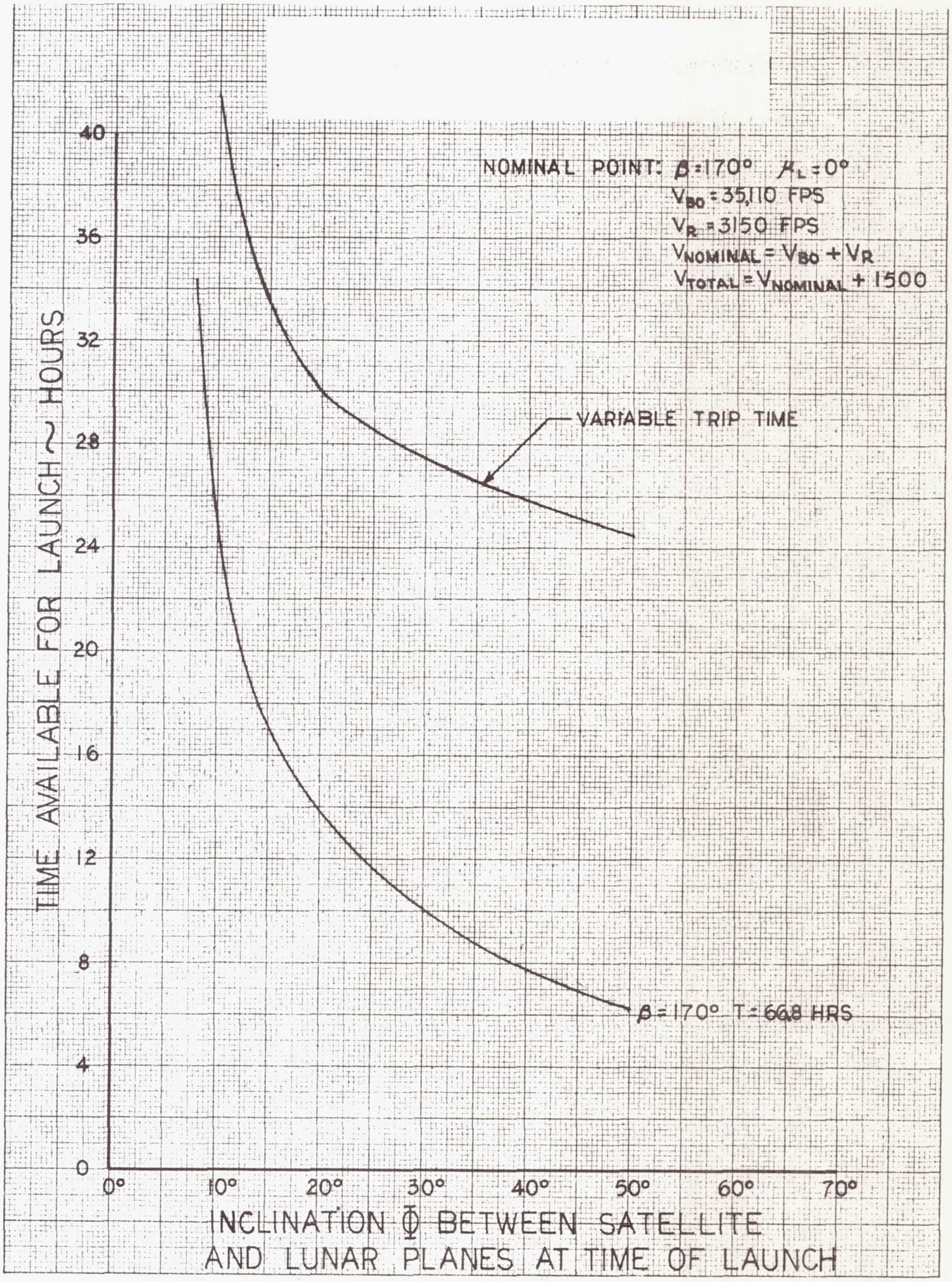


FIGURE 41 FIXED AND VARIABLE TRIP TIME ESCAPE WINDOW COMPARISON ΔV = 1500 FPS

1.6 ADJUSTMENT FOR NON-LINEAR NODAL PRECESSION

The discussion in the previous sections and related figures have assumed a linear or constant relative motion between the moon and the OLF Plane-lunar plane node line. Actually, depending on the values of i and η, the node line position, defined by Δ, is a non-linear function of λ_N This non-linearity is evident in the following analysis and can be calculated for any value of i and η.

From Figure 1, the following trigonometric relationships are obtained.

$$\cos i = \cos \eta \cos \Phi + \sin \eta \sin \Phi \cos \Delta \tag{7}$$

OR

$$\cos \Delta = \frac{\cos i - \cos \eta \cos \Phi}{\sin \eta \sin \Phi} \tag{8}$$

From spherical trigonometry

$$\frac{\sin i}{\sin \Delta} = \frac{\sin \Phi}{\sin \lambda_N} \qquad \text{OR} \qquad \sin \Phi = \frac{\sin i \sin \lambda_N}{\sin \Delta} \tag{9}$$

substituting (9) in (8)

$$\cos \Delta = \frac{\cos i - \cos \eta \cos \Phi}{\sin \eta \left[\dfrac{\sin i \sin \lambda_N}{\sin \Delta}\right]} \tag{10}$$

which reduces to

$$\tan \Delta = \frac{\sin \eta \sin i \sin \lambda_N}{\cos i - \cos \eta \cos \Phi} \tag{11}$$

Substituting the expression for $\cos \Phi$ from paragraph 1.2 in (11), the expression for $\tan \Delta$ reduces to

$$\tan \Delta = \frac{\sin i \sin \lambda_N}{\cos i \sin \eta - \sin i \cos \eta \cos \lambda_N} \tag{12}$$

From (12), with i and η constant, Δ is a non-linear function of λ_N and $\dfrac{d(\Delta)}{d(\lambda_N)}$ is not a constant. This is illustrated in Figure 42 for an i = 30° and various η's. Since the interval between window occurrence is a function of the rate of change of α_s, this interval will vary depending on i, η, and the position of the previous window. The variation in this interval between windows is illustrated in the table below for i = 30° and various η's.

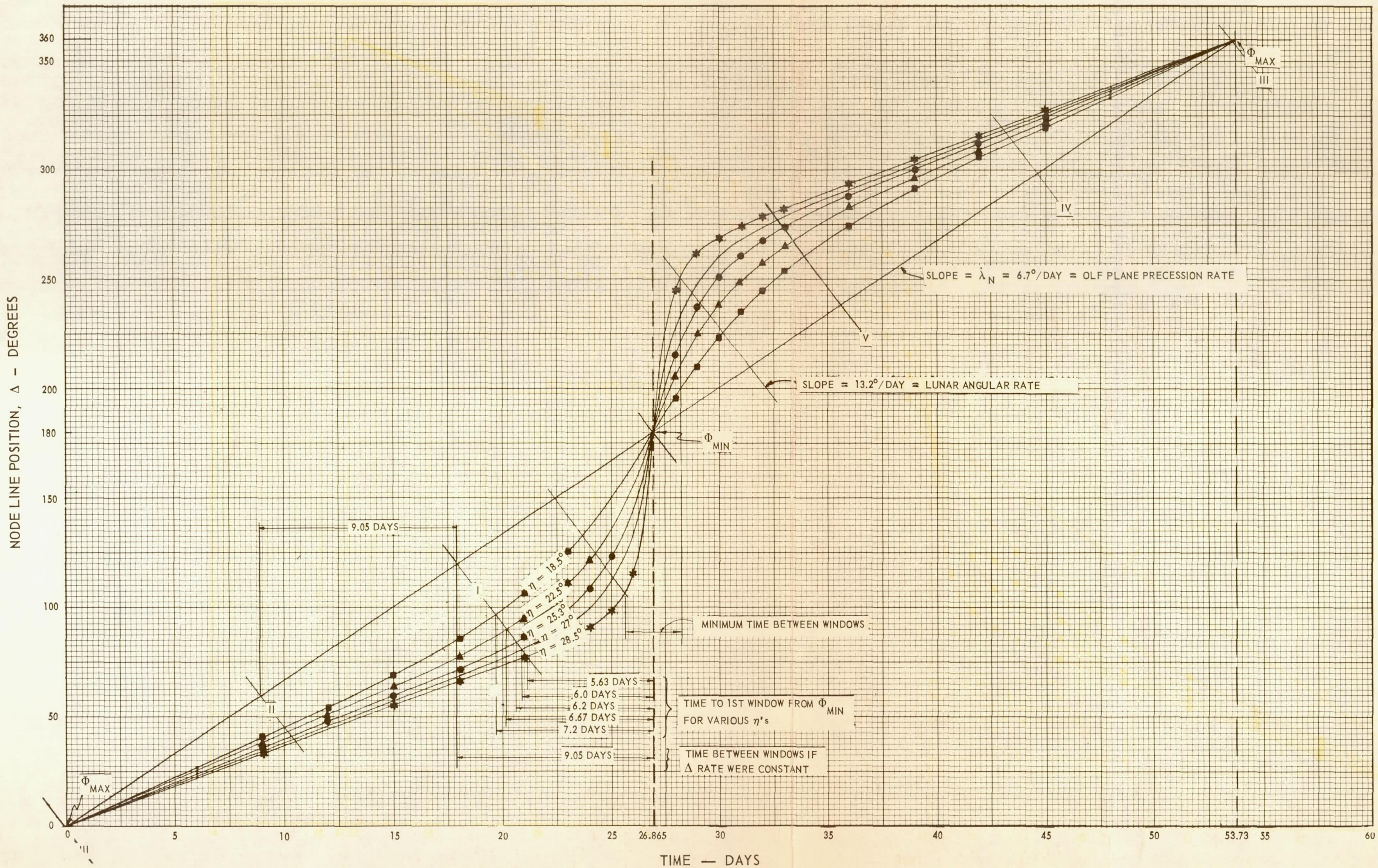


FIGURE 42 NODE LINE POSITION VARIATION WITH TIME

INTERVAL BETWEEN WINDOWS (i = 30°)

η	Window	$\dot{\alpha}_s$ = constant		$\dot{\alpha}_s$ = non-linear	
	0	0	days	0	days
28.5	1	9.05	"	5.63	"
	2	9.05	"	10.7	"
	3	9.05	"	10.6	"
	0	0	days	0	days
27.0	1	9.05	"	6.0	"
	2	9.05	"	10.5	"
	3	9.05	"	10.45	"
	0	0	days	0	days
25.3	1	9.05	"	6.2	"
	2	9.05	"	10.35	"
	3	9.05	"	10.40	"

The above table assumes the first window (window 0) occurs at the condition of ϕ minimum. It should be noted that the interval between windows can be adjusted depending on when the ϕ cycle is entered.

The effect of this non-linear geometrical condition on the window size must also be considered. Since window size is essentially dependent on both the geometric conditions as well as the vehicle constraints or allowable energy packages, any variation in the rate of change of the geometric conditions will adjust the window size. To account for this non-linearity, adjustment factor curves have been prepared for the conditions of interest and are shown in Figure 43. These curves were determined for various values of the parameter "q" = i-η. Note that the window size is most affected as both q and ϕ become smaller. Multiplication of the fixed or variable trip time windows obtained from Figures 26 through 41 by the adjustment factor results in a more precise window determination. The window adjustment is illustrated in Figure 44 in which the window sizes previously calculated for the given conditions have been modified with the factors from Figure 43.

Because of the condition of rapidly changing variables in the small q and ϕ_{min} region, the graphical technique employed for adjusting window sizes is not sufficient and any conclusive statement concerning this region cannot be made at this time. A more detailed study should be made which should include programming the window calculations on a digital computer. For the small q curves (q < 1°), the effect at small ϕ's is sufficiently large to indicate that the conditions producing these values might best be avoided and launch should occur a little before or after the ϕ_{min} condition and at values of ϕ greater than ~ 5°. This situation results from the fact that, although the geometrical size of the window is large, the rate at which the node

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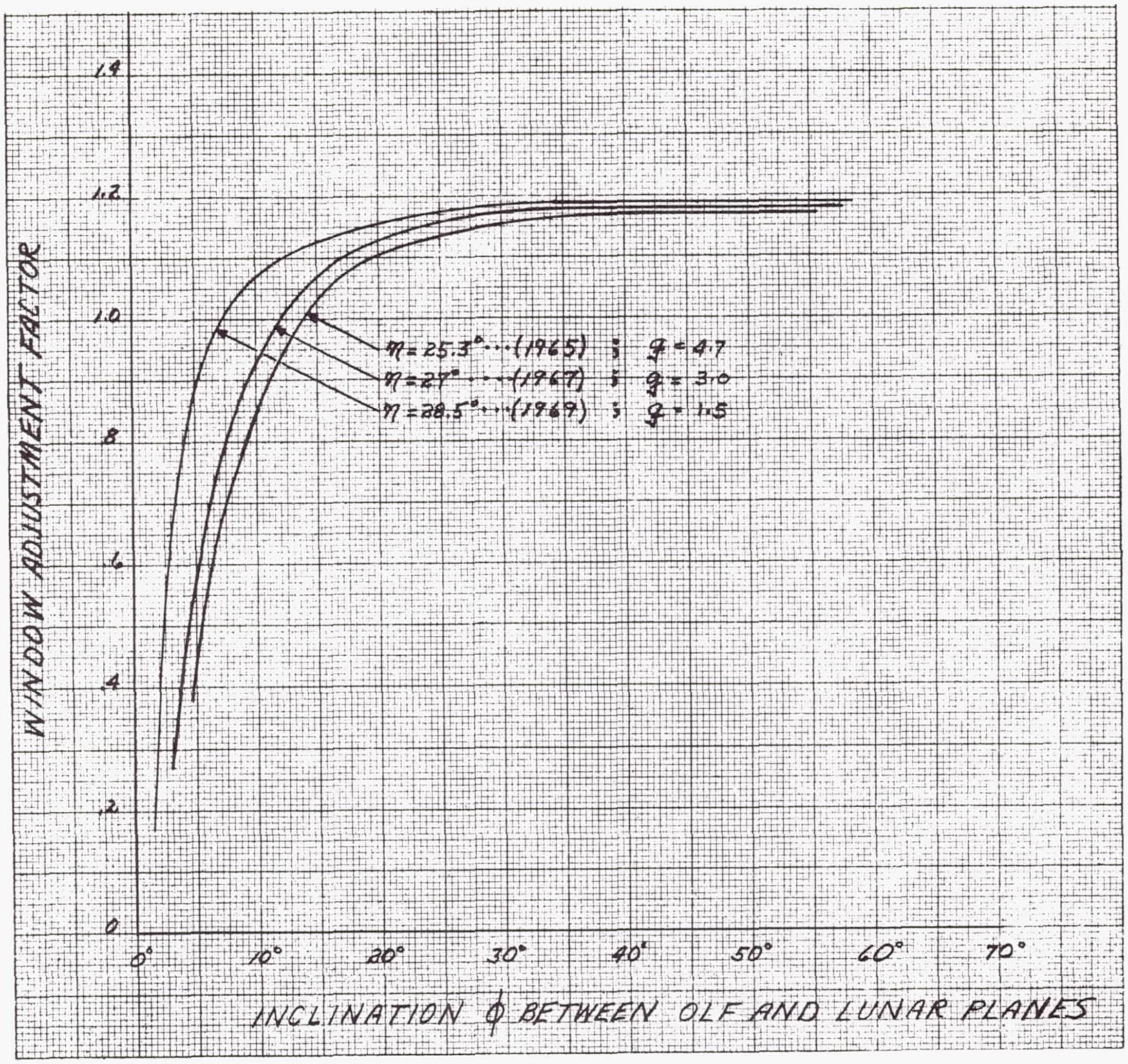

FIGURE 43 ESCAPE WINDOW ADJUSTMENT FACTORS i = 30°

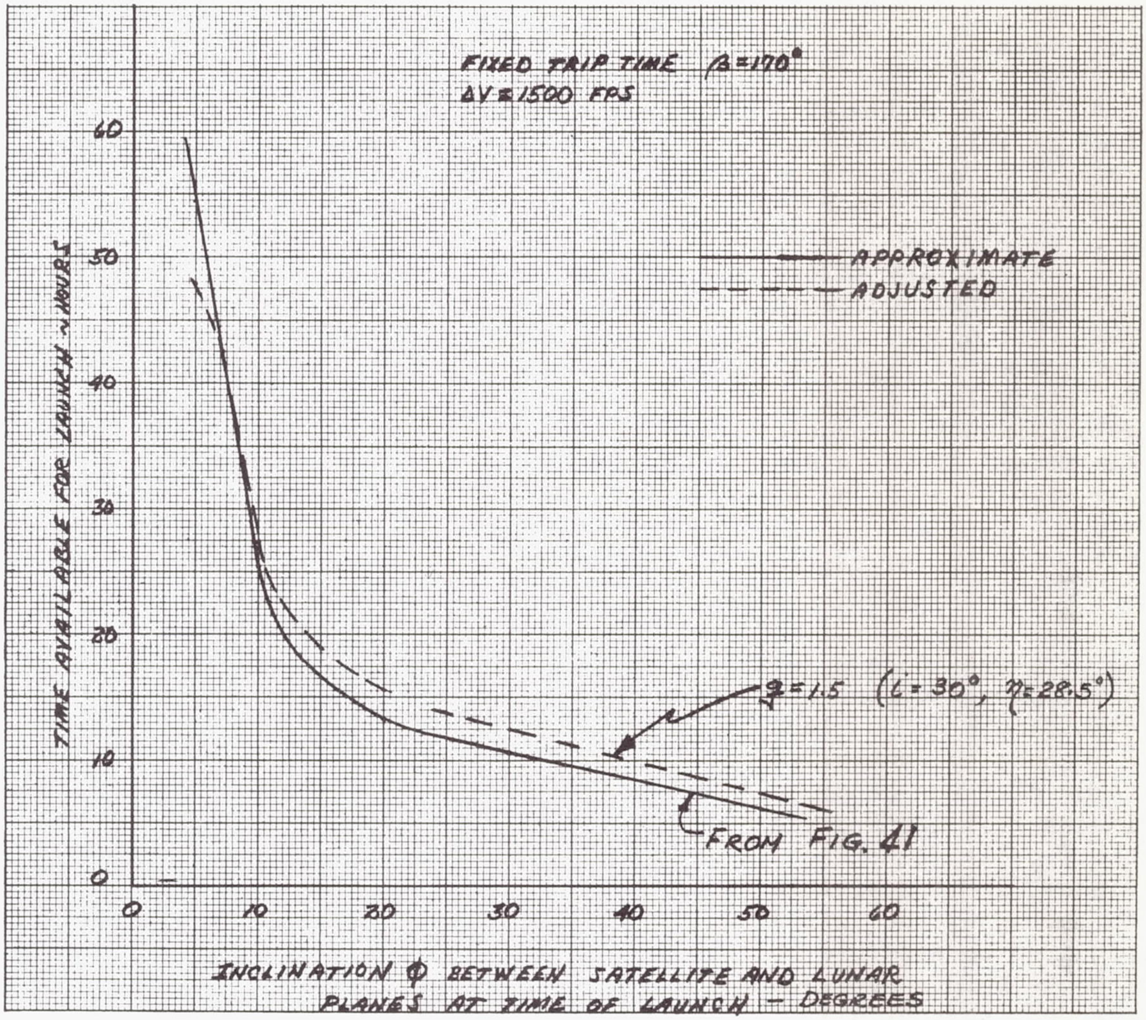

FIGURE 44 COMPARISON BETWEEN ADJUSTED AND APPROXIMATE ESCAPE WINDOWS

line passes through the window is so high as to nullify the physical condition.

A limiting condition exists when $\phi = 0$ and $q = 0$, i.e. $i = \eta$. This condition is met for $i = 28.5°$ and $\eta = 28.5°$. When this occurs, as noted before, a very large window exists because launch can be made in the lunar plane.

The effect of the non-linear motion of the node line is not altogether detrimental. By entering the ϕ cycle (establishing the OLF orbit plane) such that a launch opportunity occurs at the "knee" in the curves of Figure 42, a short interval between windows is achieved. An interval as small as 1.3 days on either side of the ϕ_{min} point can be obtained thus providing a second launch opportunity 2.6 days from the first. This is a one-time occurrence and can be utilized should the stay in orbit be limited by operational factors.

1.7 INITIAL POSITIONING OF ORBIT PLANE AND ORBITAL LAUNCH PROCEDURE

Once a satellite plane is in orbit, the rate at which the orbit precesses, and thus the rate of change of the satellite plane inclination ϕ to the lunar plane, is fixed. Although the cyclic nature of ϕ cannot be controlled once the orbit plane is established, the initial value of ϕ can be preselected by judiciously choosing the time of launch from Earth surface. This amounts to choosing the point at which the ϕ cycle (described in Fig. 9) is entered. Since for maximum escape window, it is desirable to have a small ϕ at the time of orbital launch to the moon, the initial ϕ is determined by allowing for the time T required to assemble the orbital vehicle. The ϕ cycle is then entered T hours before it reaches the minimum range. If the orbital assembly proceeds successfully, the vehicle will be checked out and ready for launch at that time when ϕ is a near minimum and the moon is positioned to coincide with the escape window. Two considerations are important here. Not only is it desired that the OLF plane be situated for minimum ϕ thus defining the <u>planar</u> relationship, but also that the moon at that time will be in the <u>proper</u> location within its plane as defined by its monthly ephemeris. The OLF plane is positioned correctly by launching at the proper time of day. The position of the moon is relatively established by launching the satellite at the correct time of the month.

Vehicle assembly would preferrably be completed early and launch would await orbit plane precession into the optimum launch position. If for some reason there is a lauch delay, then the vehicle waits for the next pass (or orbit), and the next, etc. As the delay time increases, the velocity penalty mounts. The rate of increase of this penalty is a function of ϕ and the central angle β (or trip time) which has been selected for the cislunar mission. If it is not necessary to hold β constant, the variable trip time approach may be used and the launch-on-time penalties are relieved somewhat. Once the late time becomes so large that the velocity penalty exceeds

that reserve which has been set aside for this purpose, the mission would be postponed until the OLF plane and moon position are again in position for optimum launch. However, some days later ϕ will have changed from its minimum to a higher value. For this second opportunity there will be less time available for launch (smaller launch window).

1.8 EFFECT OF CHANGES IN NOMINAL PARAMETERS

This analysis has assumed a specific altitude and inclination for the OLF as well as a nominal transfer trajectory and lunar orbit altitude. It is the intent of the following section to indicate the extent which deviations in these parameters have upon the enclosed escape window descriptions.

Figures 45, 46, and 47 show the relationship between burnout velocity, retrothrust velocity increment at the moon, central angle, and trip time for 150 and 300 nautical mile altitudes from Ref. 4. This altitude selection is representative because higher than 300 n. mi. borders on the Van Allen radiation belt while an altitude lower than 150 n. mi. enters the fringe of the atmosphere with high drag implications.

The window calculations were performed by employing the following relationships:

$$\Delta V_{total} = |V_{BO\ nominal} - V_{BO}|_{\beta} + (V_{R_{nom}} - V_R) + \Delta V_{\mu_L}$$

where

ΔV_{total} = the energy package to provide the escape window.

$(V_{BO_{nom}} - V_{BO})_{\beta}$ = the velocity difference due to a change in central angle between the selected transfer trajectory and the nominal

$(V_{R_{nom}} - V_R)$ = the lunar retrothrust velocity difference due to a change in central angle between the selected transfer trajectory and the nominal.

ΔV_{μ_L} - the velocity supplied to change planes at the time of launch.

The change in velocity with altitude for a given nominal travel time is obtained from Figures 45, 46, 47.

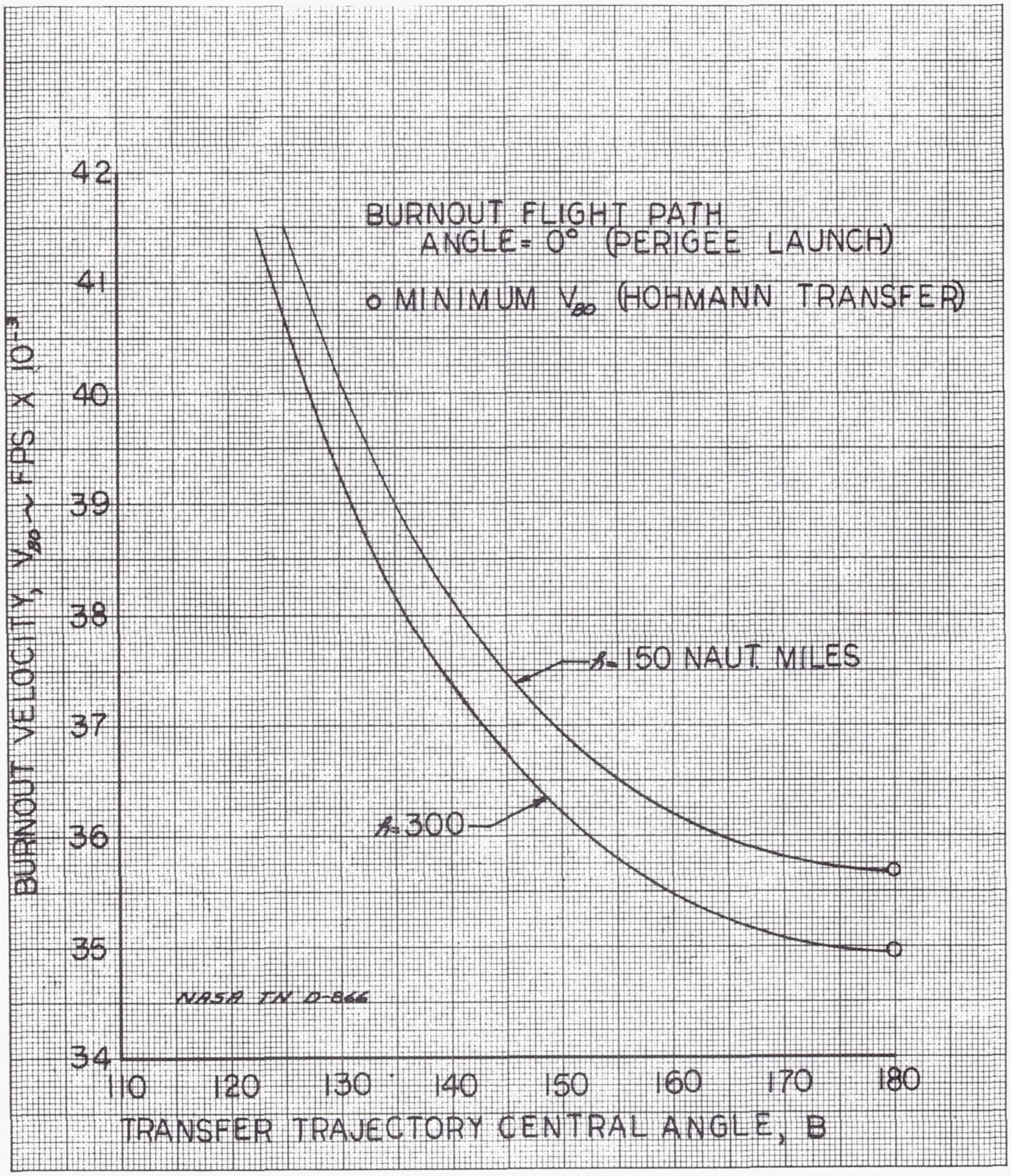

FIGURE 45 BURNOUT VELOCITY VARIATION WITH OLF ALTITUDE

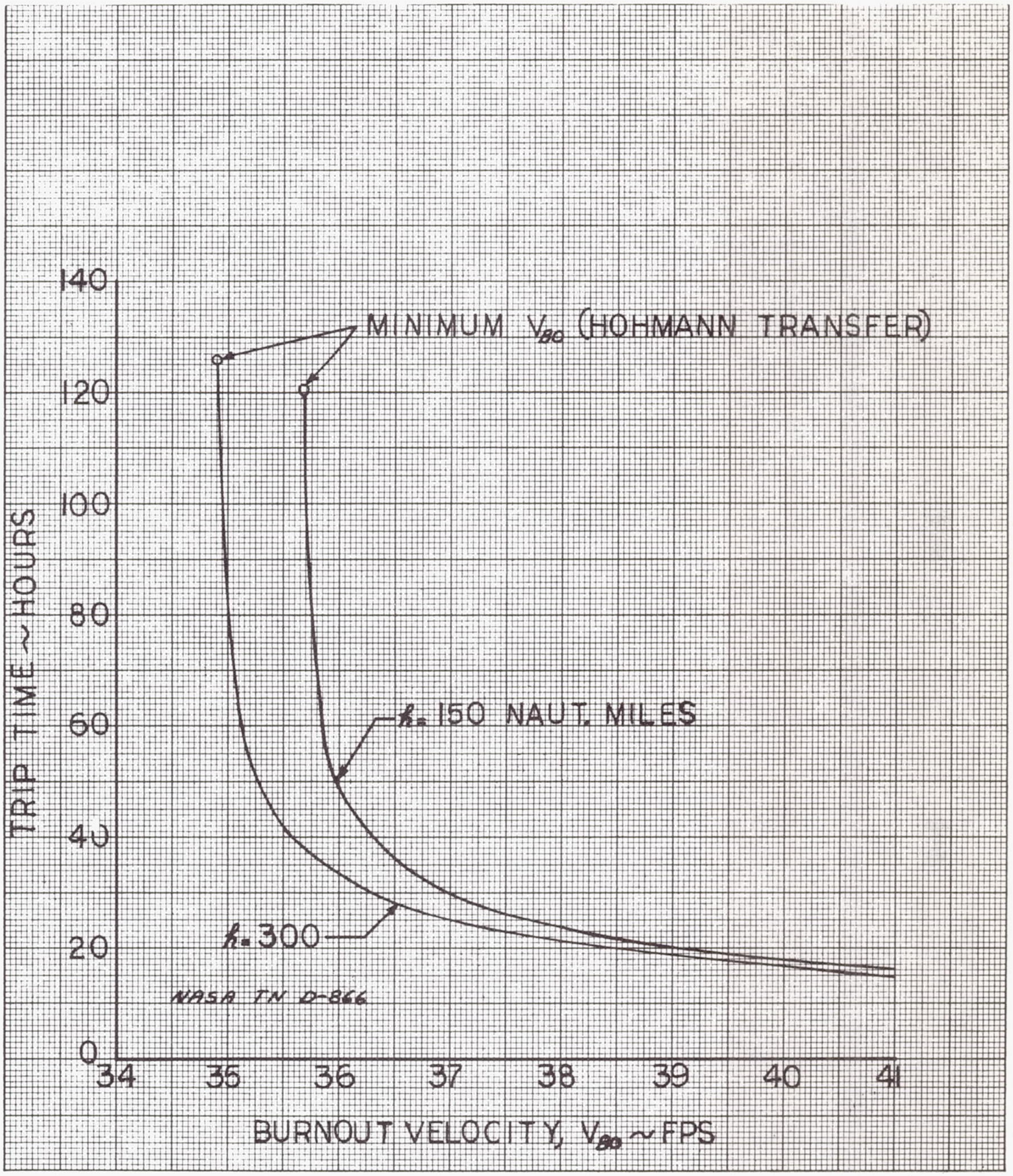

FIGURE 46 TRIP TIME VARIATION WITH OLF ALTITUDE

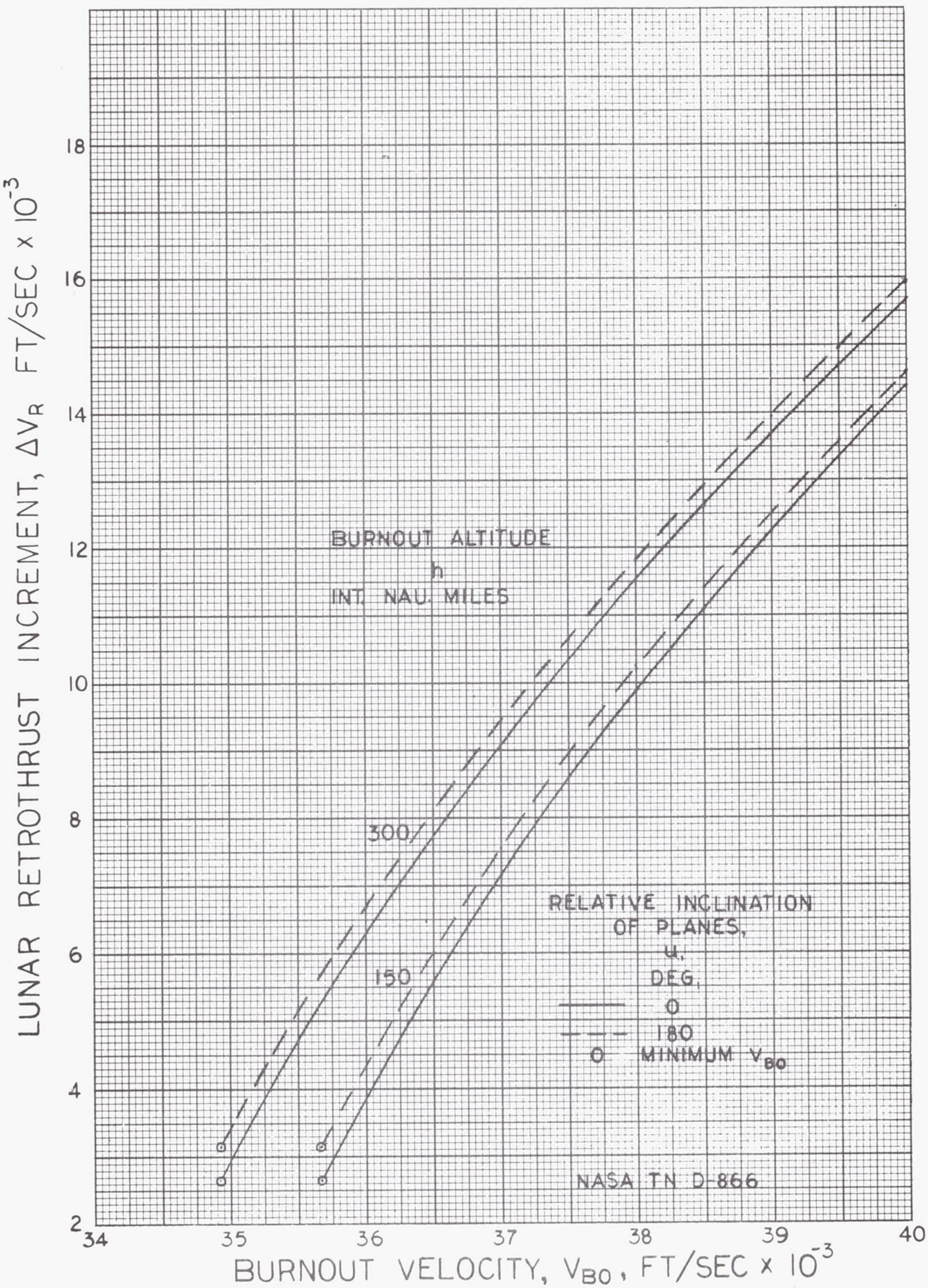

FIGURE 47 LUNAR RETROTHRUST VARIATION WITH OLF ALTITUDE

For t = 66 hrs.

h	β	$V_{BO_{nom}}$	$V_{R_{nom}}$	$V_{BO} + V_R$
150	170°	35,800	3,200	39,000
300	170°	35,070	3,200	38,270

The lower altitude mission requirement is seen to be higher by 730 fps. Thus the nominal point velocity is affected by altitude. The influence of altitude upon window size is assessed by comparing its effect upon the ΔV_{total} in the above equation. A typical 5° deviation (β = 165°) from the 170° nominal central angle is assumed.

h n.mi.	$\lvert V_{BO_{nominal}} - V_{BO}\rvert$		$\lvert V_{R_{nominal}} - V_R\rvert$		Total Difference
150	$\lvert 35,800 - 35,970\rvert$	= 170	3200 - 3800	= 600	
300	$\lvert 35,070 - 35,230\rvert$	= 160	3200 - 3750	= 550	
		10		50	60 fps

The difference in ΔV is small because the difference value between curves 45 and 46 is almost independent of V_{BO} and β. The difference in ΔV_{ω_L} for the altitudes being compared for a 10° plane change is very small and of magnitude less than 10 ft/sec. Therefore, the ΔV calculations can be generally considered independent of altitude and the escape window from ΔV and geometry considerations is nearly independent of altitude.

The above reasoning is true for a given satellite plane precession rate. However, if changes in altitude or inclination from that used in this analysis are made, adjustments to the window calculations are necessary but easily accomplished. If the precession rate is increased or decreased due to changes in the orbit parameters then the frequency with which the launch window occurs as well as the time in a given window will be increased or decreased accordingly.

The most important effect of a change in OLF plane inclination is to influence the OLF plane-lunar plane node line precession rate as discussed in paragraph 1.6.

1.9 "PUSHBUTTON ERROR"

The discussion preceding has been concerned with delays exceeding one orbit period. Of interest also is the allowable delay from the correct instant of orbital launch in any single orbit. This delay could be due to instrumentation threshold limits as well as due to random causes which would result in a launch hold beyond the instrumentation tolerances.

Since the OLF is orbiting at an angular rate (for the 263 nautical mile altitude) of 3.83 deg./min., any delay from the correct launch point would result in a decrease in the central angle β between the position vectors $\overline{L}$ and $\overline{S}$ in the transfer trajectory plane (Figure 16). Assuming that information and equipment is available to sense this change and allowance made in the burnout parameters, the effect on the transfer trajectory would be to shorten the trip time. Using a $\beta = 170°$ (T_b = 66.8 hours) as the nominal point for a correct launch, a 3 minute delay would amount to decreasing β to $\sim 158°$ with an equivalent trip time of 40 hours. In a 3 minute period, the moon's position can be considered as relatively fixed. Therefore, if launch occurred in-plane, the vehicle would arrive at the moon's orbit 27 hours ahead of the moon. Thus to intersect the moon it would be necessary to make a plane change either at launch or midcourse. If the plane change were made at launch, the change required would be a function of ϕ existing at launch. Assuming the best conditions or a reasonably small ϕ, the plane change would be about 10° for $\phi = 10°$ and less for smaller ϕ's. But with a $\phi = 50°$, the plane change would be as high as 40°.

Figure 48 illustrates the velocity penalty at various ϕ's for "pushbutton" error up to 3 minutes in any one orbit. This velocity penalty is the combined effect of the plane change requirement at launch and the resultant increase in burnout velocity from the nominal value at $\beta = 170°$.

It is apparent upon examining Figure 48, that the penalties are severe for even a delay of one minute. The alternative technique of making a plane change at some midcourse point decreases this penalty (how much depends on the vehicle design, guidance equipment, and precision trajectory data). Delaying launch to the next orbital pass is considered the most practical as well as economical approach.

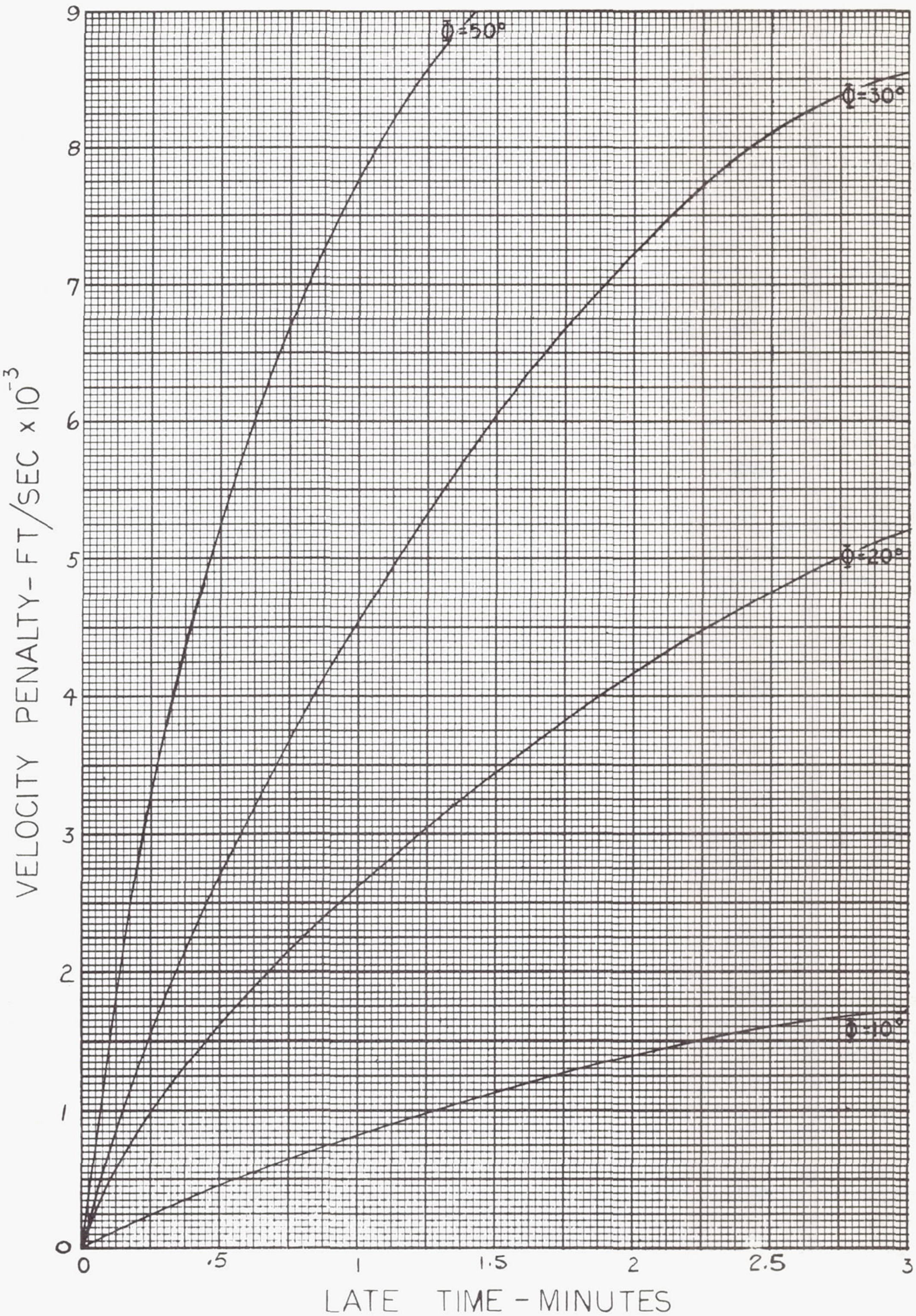

FIGURE 48 PUSHBUTTON ERROR VELOCITY PENALTY

2.0 CONCLUSIONS AND RECOMMENDATIONS

2.0.1 Conclusions

Due to the relative motion between the moon and the precessing OLF orbit plane, feasible launch opportunities (escape windows) occur several times in one month.

The interval between launch opportunities is determined by the relative positions of the OLF and lunar planes. These positions can be controlled to a certain degree by timing the first launch window occurence. An arrangement between planes for this first opportunity might be selected which provides the largest possible escape window, or the arrangement might be such that the first two escape windows follow one another in a short interval (second window occurring about 3 days after first) with the third window occurring 9 or 10 days later. As might be suspected, the largest window and short interval sequence are not synonymous but by judicious orbit plane determination, a little of both can be attained. Precise definition as to the degree which the "little of both" can be accomplished involves a detailed study in a range where the precession rate of the OLF-Lunar planes node line is extremely non-linear. The graphical techniques employed in the analysis to date are approximate but not sufficient for proper investigation of this range. A digital computer program should be used to perform the calculations accurately.

The escape window size or time available for launch is a function of three general factors.

1. Geometrical relationship between planes.
2. Extra propellant allowed for provision of the window.
3. Launch technique employed.

The first concerns the correlation between the position parameters of the moon and OLF in their respective planes and the transfer trajectory parameters. Optimum escape situations arise when the position of the moon at arrival coincides with intersection of the node line of the OLF-lunar plane at launch and the lunar orbit. The size of the window when this coincidence occurs is a function of the inclination ϕ between the two planes at the time of launch.

Since adjustments made for early or late launch involve changes in the launch and trajectory parameters which require expenditure of additional propellant, the more propellant available the longer delay times may be tolerated.

Analysis of two techniques for altering the launch conditions to insure lunar rendezvous shows the escape window can be opened considerably if a trip time tolerance is introduced. Allowing a decrease in central angle of 3° (decrease nominal trip time by 10 hours) coupled

with a 500 ft/sec. additional energy provision leads to a window several times larger than that which results from a fixed trip time restriction.

The bulk of the escape window analysis was performed for a particular OLF altitude and inclination. However, it is shown that the results are relatively independent of altitude and may be generally applied for any case within the operational orbit limits of the Orbital Launch Facility.

A list of recommended study areas related to the orbital launch escape window is included. The scope of the initial study did not permit detailed analysis of these problems.

2.0.2 Recommended Study Areas

(a) Tracking Restrictions

A cislunar shot will be tracked during the boost period, periodically during transfer for midcourse guidance correction, and near the moon for retrothrust application. Since any midcourse terminal signals would probably be sent from the DSIF net, the lunar trip should be timed to coincide with visibility from these facilities. To what extent man, plus onboard guidance equipment, can relieve these restrictions should be analyzed. The desirability (or necessity) for deep space tracking facility acquisition synchronism with the transfer trajectory also requires further investigation.

(b) Midcourse Guidance

A midcourse correction velocity package in terms of burnout errors and time after burnout at which correction occurs should be defined and analyzed. "Pushbutton" error limits as well as allowable errors in the other burnout parameters would be defined by the amount of midcourse correction fuel carried.

(c) Late Launch Correction Techniques

The present analysis assumed all corrections for late launch involved parameter adjustments at the time of launch. The merits of one gross correction at some spot in space such as the minimum velocity point have not been investigated. A study would uncover the instrumentation requirements for such maneuvers as well as their desirability from a trajectory optimization standpoint.

(d) Statistical Analysis

A practical statistical analysis of the probability for launch-on-time is necessary so as to properly assess the advantages of launching early or the use of optional holds in the launch countdown.

(e) Variable Trip Time Launch Technique

A detailed investigation of the implications of the variable trip time launch technique is necessary. This requires analysis of booster variable thrust applications, length of countdown time intervals, and mission constraints upon time of arrival at moon (such as the DSIF tracking visibility cones.)

(f) Practicality of Hohmann Transfers

These 180° central angle trajectories are avoided for two reasons:

1. Trip time from Earth to moon is too long ($\sim$ 125 hours).

2. Extreme sensitivity of miss distance at the moon to errors in burnout parameters.

If the second difficulty can be overcome by combining fine control of burnout parameters with proper design of midcourse correction, the increased trip time characteristic can be used to greatly enlarge the escape window by utilizing the variable trip time technique. Even for large ϕ values, escape windows of a magnitude of several days could be realized with small fuel reserves ($\sim$ 250 fps).

(g) Initial Powered Flight Phase

Correlation of burning phase parameters (which result in optimum boost performance) with burnout variables (which result in an optimum transfer trajectory) should be investigated. This parametric analysis would correlate burnout true anomaly and flight path angle, thrust to weight ratio, and boost burning time with the analysis already performed.

(h) Non-circularity of OLF Orbit

The effects of non-circularity of the OLF orbit have not been considered and should be investigated.

(i) Non-linear precession rate of the Olf-lunar planes node line.

The effect of this important variable upon precise window determination, particularly in the area of small ϕ and small q , should be studied in greater detail.

These study recommendations are estimated to require a minimum of twenty three man-months of engineering effort plus about 40 hours of computer time.

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